MATH 184 Computing Derivatives

Some derivative rules which you can use mechanically are:

**Sum Rule**: \( \frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x) \)

**Constant Multiple Rule**: \( \frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}f(x) \)

**Product Rule**: \( \frac{d}{dx}(f(x)g(x)) = \left( \frac{d}{dx}f(x) \right)g(x) + f(x) \left( \frac{d}{dx}g(x) \right) \)

**Quotient Rule**: \( \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{\left( \frac{d}{dx}f(x) \right)g(x) - f(x)\left( \frac{d}{dx}g(x) \right)}{g^2(x)} \)

**Chain Rule**: \( \frac{d}{dx}f(g(x)) = \frac{d}{dx}f(x)|_{x=g(x)} \cdot \frac{d}{dx}g(x) \)

We are using the notation \( h(x)|_a \) to denote the value of the function \( h(x) \) at \( x = a \). Another way to write the Chain Rule that may be easier is \( (f(g(x)))' = f'(g(x)) \cdot g'(x) \) where we have \( f' \) is \( \frac{d}{dx}f(x) \).

Thus the awkward expression \( \frac{d}{dx}f(x)|_{x=g(x)} \) is the derivative of \( f \) evaluated at \( g(x) \) (i.e. we substitute \( g(x) \) for \( x \)). We cannot directly say that \( \frac{d}{dx}x^0 = 0 \cdot x^{-1} \) since \( \frac{d}{dx}x^0 = 0 \) while \( 0 \cdot x^{-1} = 0 \) for all \( x \neq 0 \) but is not defined for \( x = 0 \). This subtle difference will show up in MATH 105 when you wish to *integrate* (or anti-differentiate) the function \( \frac{1}{x} \).

This will enable us to compute derivatives of quite complicated functions. Sometimes on tests we are only interested in *Calculator Ready* format which means that we computed a formula for the derivative but have not simplified.

We have some additional rules for special functions:

\[
\frac{d}{dx}x^p = px^{p-1} \text{ (for } p \neq 0 \text{)}
\]

\[
\frac{d}{dx}e^x = e^x
\]

\[
\frac{d}{dx} \ln(x) = \frac{1}{x}
\]

\[
\frac{d}{dx} \sin(x) = \cos(x)
\]

\[
\frac{d}{dx} \cos(x) = -\sin(x)
\]

Try to puzzle out the following. We use a mixture of the rules. I think I have chosen mostly obvious rules so that for example \( \frac{d}{dx}(g(x))^4 = 4 \cdot (g(x))^3 \cdot g'(x) \).

**Examples**

\[
\frac{d}{dx} \left( \sin(x^2 + 5)(x^3 + 3x)^4 \right) =
\]

\[
\left( -\cos(x^2 + 5) \cdot 2x \right) \left( (x^3 + 3x)^4 \right) + \left( \sin(x^2 + 5) \right) \cdot \left( 4 \cdot (x^3 + 3x)^3 \cdot (3x^2 + 3) \right)
\]

\[
\frac{d}{dx} e^x \cdot \sin(x) \cdot (x^3 + x^{1.5}) =
\]

\[
e^x \left( \sin(x) \left( x^3 + x^{1.5} \right) \right) + e^x \left( (\cos(x))(x^3 + x^{1.5}) + \sin(x)(3x^2 + 1.5x^{5}) \right)
\]

These answers are in Calculator Ready form (all derivatives have been handled) but they are very unsimplified. It is not worth simplifying in such cases.