The University of British Columbia

Midterm #1 - October 14, 2011

MATH 104

Closed book examination Time: 50 minutes

Last Name: _____ **First:** _____ **Signature:** _____

Student Number: _____ **Section Number:**_____

Special Instructions:

No memory aids or communication devices are allowed. No calculators may be used. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		6
2		6
3		8
4		12
5		8
Total		40

1. Derivatives by Definition [/6 points]

(a) Carefully state the definition of the derivative $f'(a)$ of a function $f(x)$ at $x = a$.

(b) Use the definition of the derivative and compute the derivative of

$$f(x) = \begin{cases} -4x^2 + 6x - 3 & \text{if } x < 0, \\ x^2 + 6x - 3 & \text{if } x \geq 0 \end{cases}$$

at $x = 0$. No credit will be given for the answer by any other method.

2. Computations with Derivatives [/6 points]
Use the rules of differentiation in this question. No credit will be given for the answer without the correct accompanying work.

(a) Find the derivative $f'(t)$ for $f(t) = \sqrt{t}(t^2 + e^{3t})$.

(b) Find the derivative $g'(z)$ for $g(z) = \dfrac{e^{3+z}}{z^2 + \sin z}$.

(c) Find the *equation* of the tangent line to the curve $y = (\cos(2s) + 2)^4$ at the point $s = \dfrac{\pi}{2}$.

3. Continuity and Differentiability [/8 points]
Let a, b and c denote constants.

(a) Find the values of a, b, and c so that the function

$$f(x) = \begin{cases} e^{x+a} & \text{if } x \leq -1, \\ x^3 + 2 & \text{if } -1 < x < 1, \\ c(x-1) + b & \text{if } x \geq 1 \end{cases}$$

is continuous at all points in its domain.

(b) Find the values of a, b, and c so that $f(x)$ is differentiable at $x = 1$.

4. Business Problem [/12 points]

UBC Math Inc. sells calculators that can solve any math problem for UBC students. UBC Math Inc. estimates that it can sell 40 calculators per week if it sets the price \$300 each. For every \$10 increase in price, the number of calculator sold per week decreases by 20. Assume that the fixed production costs are \$4000 and the variable costs are \$200 per calculator. In the following, use p for the price and q for the weekly demand.

(a) Find the linear demand equation for the calculator. Express p in terms of q.

(b) Find the weekly cost function $C(q)$ and revenue function $R(q)$ *as functions of* q.

(c) Find the break-even points for the calculator. Give both the price p and quantity q at each of these points.

(d) Find the marginal profit function $MP(q)$, i.e. the derivative of the profit function $P(q)$.

(e) If UBC Math Inc. is setting the price \$250 each, should it increase or decrease the price to increase its profit? Explain your answer.

(f) Use the marginal profit $MP(q)$ to estimate the additional profit incurred if the production level is increased from \$500 to \$501.

5. True or False [/8 points]

Say whether each of the following statements is true (**T**) or false (**F**) and give a reason or counterexample. This problem will be graded as follows: 2 points for a correct answer with reason or counterexample, 1 point for a correct **T** or **F** without a reason or counterexample, and 0 points for an incorrect answer. In some cases, a sketch may help you.

1. If both $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$ exist, then $\lim_{x \to a} f(x)$ also exists.

Answer:

2. The function $f(x) = \begin{cases} \dfrac{|x|^2}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$ is continuous at 0.

Answer:

3. If a function $g(x)$ is *differentiable* at a and another function $f(x)$ is *differentiable* at $g(a)$, then the composite function $f(g(x))$ is *continuous* at a.

Answer:

4. If $P(x)$ and $Q(x)$ are differentiable at a, then both $P(x)Q(x)$ and $\dfrac{P(x)}{Q(x)}$ are differentiable at a.

Answer: