

Math 184 Problem

(from Charles Lamb)

A monopoly manufacturer estimates that when the price of an item it produces is \$100 then the weekly demand for the items is 3,000. For every \$1 increase in price, the weekly demand decreases by 30 items. Assume that the fixed costs of production on a weekly basis are \$72,000 and the variable costs are \$60 per item.

- (a) Find the linear 'demand equation' for the item. Use the notation p for the unit price and q for the weekly demand.
- (b) Find the weekly linear cost function $C = C(q)$.
- (c) Find the weekly quadratic revenue function $R = R(q)$.
- (d) Find the 'break-even' points where $C = R$.
- (e) Graph $C = C(q)$ and $R = R(q)$ on the same set of axes with an eye to explaining why there are two 'break-even' points.
- (f) Find the weekly quadratic profit function $P = P(q)$.
- (g) Show that $P(q)$ on the graph in part (e) and indicate the regions of profit and loss (negative profit) on the q -axis.
- (h) How should the monopoly company operate in order to maximize the weekly profit function $P = P(q)$? Give two solutions for the correct answer.