NAME(circle last name):

Student no.:

Math 104/184 - Test 4 - Friday November 17, six pages

Show your work. No aids allowed e.g. no calculators, no cellphones etc. Simplification of answers is **not** required unless asked for in a question.

1. [30 marks] No need to simplify your final answer in these questions. Show your work.
   a) [6 marks] Compute the derivative $f'(x)$ for

$$f(x) = \frac{x}{(x^2+4)^{1/3}}$$

   b) [6 marks] Given $x^2 + y^2 = \ln(xy) + 2$, compute $\frac{dy}{dx}$ as a function of x and y.

   c) [6 marks] The function $f(x) = xe^{2x}$ has one inflection point. Find the x coordinate of this point.

d) [6 marks] Find the maximum and minimum of $g(x) = x^3 - 3x$ on the interval $[0, 2]$

e) [6 marks] We are not given an expression for the function $f(x)$ but we are given that $f(1) = 2$ and

$$f'(x) = \frac{x}{\sqrt{x^3 + 3}}.$$

Estimate the value of $f(1.1)$ using this information.

2. [15 marks] The function $f(x) = (\ln(x))^2$ is defined for $x > 0$. On what interval or intervals is the function $f(x)$ concave up?

3. [20 marks] We are producing an amount q of a chemical at a cost

$$C(q) = 50 + 2q + 2q^2.$$

Find the amount for which the average cost is minimized (justify your conclusion) and determine that average cost.

4. [15 marks] Sketch the curve for $f(x)$ defined for all x where we are given the following information. We have $f(x) > 0$ for all x and $f(1) = 1$, $f(2) = 2$, $f(3) = 3$.

$$\text{Also } f'(x) \begin{cases} > 0 & \text{for all } x \neq 2 \\ = 0 & \text{for } x = 2 \end{cases}, \quad f''(x) \begin{cases} > 0 & \text{for all } x \in (-\infty, 1) \text{ and } x \in (2, 3) \\ = 0 & \text{for } x = 1, 2, 3 \\ < 0 & \text{for all } x \in (1, 2) \text{ and } x \in (3, \infty) \end{cases}$$

and

$$\lim_{x \to \infty} f(x) = 5, \quad \lim_{x \to -\infty} f(x) = 0.$$

5. [20] You are planning on building balloons and selling their surface area for advertising. Apparently there is sufficient air travel to sell even the top surface of your balloons. You estimate the net value of a square meter of balloon surface is worth \$100 (you have computed expected revenue minus cost of balloon fabric). The cost of helium to fill your balloon is \$5 per cubic meter. You might like to remember that the area of a balloon of radius r meters is $4\pi r^2$ square meters and the volume is $\frac{4}{3}\pi r^3$ cubic meters. What is the optimal radius for your balloon? A complete answer will justify that the given radius yields the maximum profit.