MATH 104 and 184

Mock Midterm 1

26 September 2011

TIME: 50 MINUTES

FULL NAME: _____ STUDENT # : _____

SIGNATURE: _____

This Examination paper consists of 8 pages (including this one). Make sure you have all 8.

INSTRUCTIONS:

No memory aids allowed. No calculators allowed. No communication devices allowed.

MARKING:

Q1		/9
Q2		/9
Q3		/7
Q4		/5
Q5		/8
Q6		/12
TOTAL		/50

Q1   [9 marks]

Find the following limits.

(a) $\displaystyle\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$

(b) $\displaystyle\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x}$

(c) If $\displaystyle\lim_{x \to 1} f(x) = 8$ and $\displaystyle\lim_{x \to 1} g(x) = 3$, then find $\displaystyle\lim_{x \to 1} \sqrt[3]{f(x)g(x) + 3}$.

Q2   [9 marks]

Compute the derivatives of the following functions. DO NOT SIMPLIFY.

(a) $f(x) = \dfrac{x^2 + 12x + e^3}{x + e^x}$

(b) $g(t) = e^{3t}(t^2 + x^2)$

(c) $f(x) = (x^2 + x + 1)(x^3 + 1)^3$

Q3   [7 marks]

(a) (2 marks) Carefully state the definition of the derivative of a function $f(x)$ at a point $x = a$.

(b) (5 marks) Use the definition of the derivative from part (a) to compute $f'(1)$ for $f(x) = \dfrac{13}{x + 7}$. NO CREDIT will be given for any other method.

Q4   [5 marks]

Find the values of the parameters a and b such that the function

$$f(x) = \begin{cases} (2x + a)^3, & \text{if } x \leq 0, \\ 5bx + 8, & \text{if } 0 < x \leq 1, \\ x^2 + 12, & \text{if } x > 1, \end{cases}$$

is continuous at all the points in its domain. Is f differentiable at all points in its domain with these values of a and b?

Q5   [8 marks]

Find the equation of the tangent line to the curve $y = f(x) = \dfrac{1}{\sqrt[3]{x^2}}$ that is parallel to the line $y - 2x = \pi$.

Q6   [12 marks]

When EZ Electronics Company sells surge protectors at $50 a piece, they produce and sell 3000 of them per month. For every $1 increase in price, the number of surge protectors they sell decreases by 15. Assume that the fixed production costs are $50,000 and the variable costs are $30 per surge protector.

(a) Find the linear demand function $q = D(p)$, where p is a price of a unit and q is the number of surge protectors made and sold. [Hint: The point $(p, q) = (50, 3000)$ must lie on this line.]

(b) Find the cost function $C(q)$ as a function of q, and then express it as a function of p.

(c) Find the revenue function $R(q)$ as a function of q, and then express it as a function of p.

(d) Find the marginal profit, $MP(p)$, with respect to p.

(e) Find the *break-even points*. Give both the price p and quantity q at each of these points.

(f) If EZ Electronics Company is operating at the higher break-even point, should it increase or decrease the price of its surge protectors to increase its profits? Explain your answer.