

LAST NAME:

Student no.:

Math 184 - Test 4 - Friday November 16, six pages

Please show your work. I expect some arguments and, typically, correct final answers will not receive full credit if no arguments are given. If you use log indicate the base. No aids allowed e.g. no calculators, no cellphones etc.

a) [6 marks] Compute the derivative of

$$f(x) = e^{\cos(x^2)}$$

b) [6 marks] Find the maximum and minimum of the function $f(x) = 2x^3 - 3x^2$ on the interval $[-1, 2]$.

c) [6 marks] Given $xy^2 + e^{xy} = 5$, determine $\frac{dy}{dx}$ in terms of x and y .

d) [6 marks] Find where the function $f(x) = \frac{1+\ln(x)}{x}$ is increasing.

e) [6 marks] Let

$$f(x) = 9(x)^{\frac{5}{3}}$$

Compute $f''(8)$ as an integer.

2. [15 marks] Rather than be given $f(x)$ explicitly, we are given that

$$f'(x) = (x^2 + 2x)e^{-x}.$$

For what values of x is the function $f(x)$ concave up?

3 [20 marks] We are producing textbooks for sale at a Southwest Canadian University. The cost of producing q text books is

$$C(q) = \frac{-100}{q} + 2 + 5q$$

Find the number of textbooks which minimizes the average cost per text. For that quantity q , also compute the marginal cost and the average cost.

NOTE: There was an error in the nature of the function $C(q)$. Either you should try to maximize the average cost (somewhat odd!) or perhaps try a well behaved function such as $500 + 2q + 5q^2$.

4 [15 marks] Sketch the curve $y = f(x)$ given the following information. Your curve should demonstrate how the curve is shaped and you should identify the interesting features e.g. inflection points, asymptotes etc.

You are given that the domain is all $x < 5$. $f(0) = 4$, $f(2) = 2$, $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow 5^-} f(x) = -\infty$

$$f'(x) \begin{cases} < 0 & \text{for } 0 < x < 5 \\ = 0 & \text{for } x = 0 \\ > 0 & \text{for } x < 0 \end{cases}, \quad f''(x) \begin{cases} < 0 & \text{for } -1 < x < 1 \text{ and } 3 < x < 5 \\ = 0 & \text{for } x = -1, 1, 3 \\ > 0 & \text{for } x < -1 \text{ and } 1 < x < 3 \end{cases}$$

5. [20 marks] You are optimizing costs for a trucking firm. You are about to send a truck for a 1000 km trip. If the truck is driven at a constant speed of x km/hr, the fuel cost in \$ per km is

$$.50 + .01x$$

The labour cost for the driver is \$25 per hour. (Note: this is not \$ per km.)

a)[1 mark] To minimize the fuel cost, what speed x would you wish to choose?

b)[1 mark] To minimize the labour cost, what speed x would you wish to choose?

c) [18 marks] For what speed x is the total cost of the 1000 km journey minimized?