

LAST NAME:

Student no.:

Math 184 - Test 3 - Friday October 26, six pages

Please show your work. I expect some arguments and, typically, correct final answers will not receive full credit if no arguments are given. If you use a logarithm, indicate the base. No aids allowed e.g. no calculators, no cellphones etc.

1. [24 marks] Compute the derivatives of the following functions. No need to simplify except in d).

a) [6 marks]

$$f(x) = e^{\sin(x)} + 5$$

b) [6 marks]

$$f(x) = x^x$$

c) [6 marks]

$$f(x) = x \ln(\cos(x))$$

- d) [6 marks] Compute $f'(5)$ as an integer given $g(5) = 10$, $g'(5) = 5$ and where

$$f(x) = \frac{(g(x))^2}{x}$$

2. [8 marks] For what x does the graph of $y = e^{3x} + e^{-2x}$ have slope zero.

3 [8 marks] Compute $\frac{dy}{dx}$ in terms of x and y when given

$$y + \ln(y + 3) = x^3 + 5$$

4 [14 marks] A curve is given implicitly by the equation

$$xy^2 = 12$$

We note that $(x, y) = (3, 2)$ is on the curve. Determine the tangent line to the curve at the point $(3, 2)$. Then use this tangent line to approximate the positive value of y when $x = 3.1$

5. [14 marks] Consider the Newton-Raphson method applied to $f(x) = x^3 + 3x + 3$. Start with $x_0 = 0$ and compute x_1, x_2 . Report x_2 as a fraction.

6. [16 marks] A firm has discovered that the relation between p and q is given implicitly by the equation

$$p^2 + q^3 = 64$$

- a) [3 marks] What are the ranges on p and q that would be imposed?

- b) [6 marks] Compute $\frac{dp}{dq}$ in terms of p, q .

- c) [7 marks] Given a cost function $C(q) = 100 + 2q$ and a profit function $P(q) = pq - C(q)$, determine $\frac{d}{dq}P(q)$ in terms of p and q alone.

7. [16 marks] A 10m ladder (not an extension ladder!) is leaning up against a vertical wall with the top of the ladder currently 8m from the base of the wall. The ladder's top is slipping down the wall at a rate of 3 m/minute while the bottom of the ladder slips away from the base of the wall.

a) [10 marks] How fast is the bottom of the ladder slipping away from the base of the wall?

b) [6 marks] Consider the product of the following two distances: the distance of the top of the ladder from the base of the wall and the distance of the bottom of the ladder from the base of the wall. What is the rate of change of this product currently as the ladder slips?