Social choice / Voting

**Majority**: select A if most voters prefer A to any alternative.

Happens if only 2 options, otherwise might not.

**Plurality**: Each voter picks one option. Elect the option with most votes. (First past the post).

<table>
<thead>
<tr>
<th>Thunder Bay</th>
<th>15,870</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lakehead</td>
<td>15,302</td>
</tr>
<tr>
<td>The Lakehead</td>
<td>8,377</td>
</tr>
</tbody>
</table>

Premise: each voter has a ranking of candidates

$A >_i B$  $A$ preferred over $B$ by voter $i$

$>_i$ complete order on candidates
Voting rule: given \( (>i) \) for each \( i \), pick one.

Ranking rule: given \( (>i) \) determine social rank \( \succ \).

\( A \succ B \): society prefers \( A \) to \( B \).

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Given a voting rule, can get a ranking:

- Winner of vote is 1st.
- Remove winner from all list.
- Winner of new lists is 2nd, etc.

Given ranking rule, one voting rule is "take top option" in \( D \).

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Runoff: If no maj. in first round, 2 candidates with most votes are chosen in round 2.

Borda Count: Each voter gives \( N \) pts to 1st option, \( (N-1) \) to 2nd, \( (N-2) \) to 3rd, etc.
Dictatorship: One voter decides.

Anonymity: Identity of voters irrelevant. If orders $\geq_i$ are exchanged between voters, outcome is the same.

Unanimity: If $\forall i \ A \geq_i B$ then also $A \geq B$

Condorcet Winner: Candidate A who wins 1-on-1 against every other candidate.

E.g. 3 candidates 

30% $A \geq B > C$
45% $B > C > A$
25% $C > A > B$

Plurality: B wins.

Runoff: AB in round 2, A wins.
Strategic voting: Improve outcome by lying about $\succ_i$.

$\exists$ voter $i$, $\succ'_i$ (another order) s.t. if voter $i$ changes from $\succ_i$ to $\succ'_i$, then outcome is better for voter $i$.

In e.g. with runoff: If 10% change vote from $B > C > A$ to $C > B > A$: $A$ eliminated, $B, C$ in round 2. $B$ wins.

Instant runoff: On the ballot, each voter ranks the candidates.

In each round, eliminate the candidate with fewest 1st place votes.

(aka transferable vote).

Monotonicity: If $A$ moves up in all orders $\succ_i$ to $\succ_i'$, then $A$ should move up in $A < B < C < D$. 