Kuhn Poker Variation

Deal

P1:

P2:

P1

±2 +1 ±1

±3 -1

call Fold 2 Bet 2 Pass
call Fold
$A^H = (1, 0), \ A^T = (0, 1)$

Bob's strategy:
for $n=2$

- Bob's first action

$\frac{1}{2}$ — same if Alice picked $T$.

$p(Bob\ picks\ H\ if\ Alice\ picked\ H\ in\ round\ 1)$

For 3 rounds:

Tree of actions depends on Alice's prev. moves.

Note: You may assume that Bob's choices do not depend on his previous actions.
Repeated games

\[ \begin{array}{c|cc}
 & C & D \\
\hline
C & (6, 6) & (0, 8) \\
D & (8, 0) & (2, 2) \\
\end{array} \]

Prisoner's dilemma variation.

Play repeatedly. $\beta$ discount ratio, $\beta \in (0, 1)$

If in game $n$ you get $M_n$, total value is $\sum_{n=0}^{\infty} \beta^n M_n$.

If $\beta < 1$: Best you can hope from games 1, 2, ... is $8\beta + 8\beta^2 + 8\beta^3 + \ldots = 8 \frac{\beta}{1-\beta}$

If $\frac{8\beta}{1-\beta} \leq 2$, may defect in game 0.

I.e. $\beta \leq \frac{1}{5}$
Claim: for $\beta$ large enough, tit-for-tat is a N.E.

If both play this, payoff is $\sum_{n=0}^{\infty} 6 \cdot \beta^n = \frac{6\beta}{1-\beta}$

Assume opp. uses T.F.T., can we do better?

Use seq. Us, opp

\[
\begin{array}{c|cccccccc}
& C & C & D & D & D & D & C & C \\
C & C & C & D & D & D & D & C & C \\
\text{opponent's prev. action} & C & C & D & D & D & D & C & C \\
\text{our payoff} & 6 & 6 & 8 & 2 & 2 & 2 & 2 & 0 & 6 & 6 & 6 \\
\text{(relative to all C) difference} & 0 & 0 & 2 & 4 & 4 & 4 & 4 & 6 & 0 & 0 & 0 & 0
\end{array}
\]

Total gain: $2\beta^i - 4\sum_{t=i+1}^{j-1} \beta^t - 6\beta^i = 2(\beta^i + 2\beta^j) - 4 \sum_{t=i+1}^{j} \beta^t$
Gain = \(2(\beta^i - \beta^j) - 4 \frac{\beta^i - \beta^j}{\sqrt{\beta} - 1}\)

\[= (\beta^i - \beta^j)[2 - \frac{4}{\sqrt{\beta} - 1}] < 0 \text{ if } \beta > \frac{1}{3}\]

If \(\beta > \frac{1}{3}\), switching to \(D\) in rounds \([i - j]\) costs us, so any sequence of actions is worse than all \(C\).

\[P1: CDCDCCD\quad P1 \text{ gets } (6, 8, 6, 8, \ldots)\]

\[P2: CCCCCCC\quad P2 \text{ gets } (6, 0, 6, 0, \ldots)\]

\[\text{avg is } (7, 3)\]

\(P1\) strategy: alternate \(C,D\), if \(P2\) defects:

always defect afterwards.
If $p_2$ does not $c$ always, they get on avg. $\leq 2$.

$p_2$: I'll coop. while $p_1$ uses $C CCCD$...

If $p_1$ deviates from this, I'll defect always.

This gives a N.E with long-term avg. payoff $(7,3)$.

If $p_1$ plans $C CCCD$ repeated, $p_2$ gains by always $D$. 