More on partial information:

Play one of two games, both 0-sum.
Alice (P1) knows the game, Bob does not.

\[ A = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \quad A' = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \]

Alice can get 0.

Now we play same game twice. (same coin toss)

Alice can still get 0, no more.

Version 2: Same rules, except \[ A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad A' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \]

If played once: payoff is \( \frac{1}{2} \).

in round 1 Alice can get \( \frac{1}{2} \) by using her info.

\[ \Rightarrow \] Bob pays 0 in round 2. Total = \( \frac{1}{2} \).

Alice can achieve \( \frac{3}{4} \) by playing randomly in round 1, use info. in round 2.
This is a 0-sum game. If value is \( v \), then Bob has a strategy giving \( < v \) no matter what Alice does.

Bob's \( \frac{3}{4} \)-strategy: In round 1 choose randomly.

In round 2: pick opposite of Alice 1st move w.p. \( \frac{3}{4} \) same

Avg payoff is \( \leq \frac{3}{4} \) no matter what Alice does.

Q: what if repeated more times?
Donation game:
each player has 2, can cooperate: give 2, other gets 6
defect: keep 2, other gets 0.

\[
\begin{array}{c|cc}
\text{C} & \text{P} \\
\hline
\text{C} & (6,6) & (0,8) \\
\text{D} & (8,0) & (2,2)
\end{array}
\]

Only N.E. is to defect, for both.

Play this repeatedly.

If played fixed number of times, T, then only N.E.
is to always defect.

(Induction: in last round always defect.)

Non fixed horizon: play \( \infty \) rounds.

\( \text{Payoff is } \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} A_{it} i_t \) \text{ [limit of avg payout]}

Discounted payoff
Discounted payoff: If in round $t$ you get $M_t$, payoff is $\sum_{t=1}^{\infty} \beta^t M_t$ for some $\beta \in (0,1)$.

$\beta \ll 1$ ⇒ very greedy. Only early rounds matter.
$\beta \approx 1$ ⇒ long horizon.

Pure strat: what do I do, given all previous games?
mixed strat: what dist. on \{C, D\}, given all past for each round.

Tit-for-Tat: In round 1 ⇒ C
In round $n>1$ : mirror opponent from prev. round.