Hex

Take turns claiming hexagons.

Goal: create a path connecting your two sides.

Blue wins.

either

or

Red wins

Blue wins
Theorem: In the \( n \times n \) Hex game, Player 1 can win.

Proof: Strategy stealing.

Assume P2 has a winning strategy. P1 makes some first move (arbitrary) say at \( x \).

Follow strategy for P2, ignoring \( x \).

P2 cannot use \( x \).

If P1 needs \( x \), play arbitrary \( x' \).

So P1 will win. \( \square \)
Bridge-it: Strategy stealing $\Rightarrow$ 1st player can win.

explicit strategy: Based on two trees.

when red removes an option, re-attach the tree. At all times, leave connected green + blue trees.

Eventually, end up with a connection Top-Bottom. A connected: Can reach any vertex to any other, using only black/blue edges.
In random turn hex on \( n \times (an) \) board,\n
\[ P(Blue \ wins) \xrightarrow{n \to \infty} F(a) \]

In random turn Bridge it \( an \) same holds.
Alice, Bob play a partisan game.

In each position:

- If Alice 1st \( \Rightarrow \) Alice wins
- If Bob 1st \( \Rightarrow \) Bob wins

N- pos.: 1st player wins
P- pos.: 2nd wins
A- pos.: Alice wins
B- pos.: Bob wins.

e.g. Subtraction game. Alice can take \( \leq 4 \) chips
Bob
\( \leq 3 \) chips
From $x$:
If Bob can move to some P or B pos., then $x$ is N, or B
otherwise $x$ is P or NA.

If Alice can move to some P or A pos. then $x$ is A or N
otherwise, $x$ is P or B.

e.g. subtraction: $S_A = \{1, 4\} \quad S_B = \{2, 3\}$

$\begin{array}{cccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\
\hline
\end{array}$

(repeats)