Theorem: The Grundy value of \((x_1, \ldots, x_k)\) in a sum of \(k\) games is the nim-sum \(g(x_1) \oplus \cdots \oplus g(x_k)\) in the single games.

e.g. Domineering:

<table>
<thead>
<tr>
<th>Board</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

- : followers are 

- followers: \{ \} or 

- followers: \{ \} or \{\} or \{\} or 

\(g(\text{ }) = 3\)
Proof: need 2 things.

1. every value < \(g(x_1) \oplus \ldots \oplus g(x_k)\) is \(g(x_1, \ldots, y_i, \ldots, x_k)\) for some follower, and
2. no follower has same value.

Assume the thm holds for all followers, so

\[ g(x_1, \ldots, y_i, \ldots, x_k) = g(x_1) \oplus \ldots \oplus g(y_i) \oplus \ldots \oplus g(x_k) \]

\[ = g(x_1) \oplus \ldots \oplus g(x_i) \oplus \ldots \oplus g(x_k) \]

since \(g(x_i) \neq g(y_i)\). Claim 2 follows.

For 1: need that can reduce some \(g(x_i)\) to get any desired sum.

\(g(y_i)\) can take any value \(0, \ldots, g(x_i) - 1\).

This is almost identical to the proof for NIM.
Bridge-it

XXX

O O O
Partisan games: players have different moves, goals.

Theorem: In Bridge-it, one player must win. (no draw)

Proof: continue till the grid is complete.

Enter in A/B, exit Y/Z.

Two options:

\[
\exists L-R \text{ crossing } \quad \exists T-B \text{ crossing.}
\]

Can both players win?