Inclusion-Exclusion

\[ P(E \cup F) = P(E) + P(F) - P(E \cap F) \]

proof

\[ E \cup F = A \cup B \cup C \]

where \( A = E \setminus F \), \( B = E \cap F \), \( C = F \setminus E \)

\[ P(E) = P(A) + P(B) \]
\[ P(F) = P(B) + P(C) \]
\[ P(E \cup F) = P(A) + P(B) + P(C) = P(E) + P(C) \]
\[ = P(E) + P(F) - P(B) \]

\[ \Box \]
RECALL!

Select $k$ out of $n$ elements unordered, no replacement, the number of ways is \[ \binom{n}{k} = \frac{n!}{k!(n-k)!} = \binom{n}{k} = \binom{n}{n-k} \]

\begin{equation*}
\text{binomial coefficient}
\end{equation*}

**Binomial Theorem**

\[ (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \]

Proof: \[ (x+y)(x+y)(x+y) \ldots (x+y) = x \cdot x \ldots x + x \ldots x \cdot y + \ldots \]

pick $x$ or $y$ from each $(x+y)$

number of terms equal to $x^k y^{n-k}$ is \( \binom{n}{k} \). \qed
Note: \( \binom{n}{k} = 0 \) if \( k > n \)

\[
\binom{n}{0} = \binom{n}{n} = 1
\]

\[
\binom{n}{k} = \binom{n}{n-k}
\]

Summary: To select \( k \) out of \( n \):

- **Ordered with replacement:** \( n^k \)
- **Ordered without replacement:** \( \frac{n!}{(n-k)!} \)
- **Unordered:** \( \binom{n+k-1}{k} \)

VPC, VCP, PVC, PCV, CVP, CPV

VPP, PVP, PPV
eg: How many ways to select 3 men + 3 women from 40 men + 50 women?

Ans: \( \binom{40}{3} \cdot \binom{50}{3} \)

eg: If select 6 at random, \( P(\text{get 3 men}) \)?

Ans: \( \frac{\binom{40}{3} \cdot \binom{50}{3}}{\binom{90}{6}} \)

eq: what is \( P(\text{full house}) \) in 5 card poker hand?

Ans: \( S = \{5\text{-card hands}\} \)

\( |S| = \binom{52}{5} \).

\( E = \{\text{full house}\} \)

\( |E| = 13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2} \).
\[ |E| = 13 \cdot 12 \cdot \binom{4}{3} \cdot \binom{4}{2} \]

*Always make sure, E in same category as S.*

E.g. both ordered / not...

Q: what is \( P(\text{full house}) \) in 5-dice game?

\[ \rightarrow 3 \text{ equal dice, 2 equal dice: } aabb \text{ in any order} \]

Ans: \[ |S| = 6^5 \] \[ |E| = 6 \cdot 5 \cdot \binom{5}{3} \]

\( a \text{ value} \) \[ |E| \text{ value} \] \[ \text{which dice are} \]

This is ordered!
If use $n$ sided die: $n(n-1)\left(\frac{5}{3}\right)$