Recall:

Impartial Combinatorial Game \([ICG]\)

Both players have same moves allowed, no randomness/chance, full information.

Progressively bounded: A position \(x\) there is a bound \(B(x)\) s.t. starting at \(x\), the game last \(\leq B(x)\) moves.

\[ N_i = \text{positions where next player can win in } \leq i \text{ moves}, \]

\[ P_i = \text{"previous" } \leq i \]

\[ N = \bigcup_{i=0}^{\infty} N_i, \quad P = \bigcup_{i=0}^{\infty} P_i. \]

Strategy: a choice of move from every pos. in the game.
Theorem: In a progressively bounded game [ICG], every pos. is in N or in P.

Rules: If all followers in N, x e P.
If some follower in P, x e N.

E.g., subtraction \( \{1, 3, 4\} \)

Theorem: In subtraction game with set S finite, the type seq. is eventually periodic.

Proof: Assume \( \max(S) = k \).
Type of N determined by types of \((n-k, ..., n-1)\).
This vector has \(2^k\) possible values.
It repeats some time!
Type of $n-k$ same as $m-k = n+t-k$  
$m= n+t$
$m-k+1 = n+t-k+1$

$m-1 = n+t-1$

Then \( \text{Type}(n) = \text{Type}(m) \)

By induction, \( \text{Type}(n+i) = \text{Type}(m+i) \) \( \forall i \geq 0 \)

= Type\(m+t+i\)

i.e. Period $t$. \( \square \)
**Theorem**: An $n \times m$ board of Chomp is an $N$ pos.

[1st Player Can Win]

**Strategy Stealing**

Assume (for contradiction) Player 2 can win.

Every follower of $(n \times m)$ is $N$ pos.

Has $1 \times 1$ has $N$ so has a $P$-follower

But this is also a follower of $[n \times m]$ so $[n \times m]$ is not $P$. 