Chomp (Example 1.0.1)

Rules of chomp

- Two players

- In their turn, a player:
  1. Chooses one eaten chocolate
  2. Removes the rectangle of chocolate: pieces above and right of the chosen one.

- The loser is the player who eats the broccoli.
Chomp 2 x 3

Losing position for the next player:

Which positions lead to the brocoli?

Winning positions for the next player:

... continue
Claim: Chomp is progressively bounded.

Proof: If the chocolate has \( N \) squares, then the maximal number of moves until we reach a final position is \( N \) because at each turn a player eats at least one piece of chocolate.
Definitions

Elements of a combinational game:
* Two players
* A set of positions $\times$ (including a subset of terminal positions)
* A set legal moves between them.

A combinational game is progressively bounded if, for every $x \in X$, there exists a finite bound on the number of moves until the game finishes.
A combinatorial game is impartial if the winning positions and available moves are the same for both players. Otherwise, it is a partisan game.

Eg: Impartial: Chomp. Partisan: Chess.

We are interested in impartial, progressively bounded games.
Substraction. (Example 1.1.1).

* Two players
* Start with a pile of $x \in \mathbb{N}$ chips
* In each turn a player takes $1$ to $4$ chips
* The winner is who removes the last piece.