Problem 1. Analyse 3-card Kuhn poker (problem 6.4 from Peres-Karlin).

- There are 3 cards: Jack, Queen and King.
- Each player pays $1 into the pot.
- Each player is dealt one of the cards.
- Player I can either pass (P) or bet (B) $1.
  - If player I bets, then player II can either fold (F) or call (C) (adding $1 to the pot).
  - If player I passes, then player II can pass (P) or bet $2 (B). If player II bets, then player I can either fold or call (match the bet).
- If one of the players folds, the other player takes the pot. If neither folds, the player with the high card wins what’s in the pot.

Find a Nash equilibrium in this game via reduction to the normal (matrix) form. Observe that in this equilibrium, there is bluffing and overbidding.

Edit. In the version above, if player 1 passes, player 2 can either pass, or bet $2. Previously this was $1.

Hints. Some strategies are obviously dominated and can be removed: as first player, never call a bet with the Jack, always call with the King. For Player 2, the strategy says what to do in each of 6 cases, depending on the card they have and whether Player 1 passed or bet.

Problem 2. Consider the game from class:

- A coin is tossed, and Alice sees the result.
- There are $T$ rounds (with the same coin).
- In each round, Alice picks H or T and Bob picks H or T, and their choices are revealed.
- At the end of $T$ rounds, Bob pays Alice 1 for each round where they both picked the same as the hidden coin.

As noted in class, the value for $T = 1$ is $1/2$ and for $T = 2$ it is $3/4$. What happens for 3 rounds?
  (a) Show that Alice can guarantee getting at least 1 on average.
  (b) Find a strategy for Bob where he pays at most 1 on average.

Problem 3 (bonus). What happens for higher $T$ in the previous problem? Let $V_T$ be the value. A full solution is to calculate the value with proof. Partial credit for non-matching lower and upper bounds on the value. The following are some directions to consider.
  (a) What is $V_4$? Compute the value if you can. Give upper and lower bounds if not.
  (b) Prove that $V_T \geq V_{T-1} + \frac{1}{4}$, and therefore $V_T \geq \frac{T+1}{4}$. 
(c) Prove that for $T$ even, $V_T \leq \frac{3T}{8}$.
(d) Prove that $\limsup \frac{V_T}{T} \leq \frac{1}{3}$.
(e) Prove that $\limsup \frac{V_T}{T} = \frac{1}{4}$.

**Problem 4.** Might be added after Monday’s class.