Mathematical Game theory
Assignment 1, due 2019-09-13

Problem 1. Consider the subtraction game.
(a) Find the set of P-positions for normal winning condition with subtraction set \{1,2,5\}.
(b) Find the set of P-positions for the misere winning condition with subtraction set \{1,2,5\}.
(c) Find the set of P-positions for normal winning condition with subtraction set \{1,4,5\}.
Justify your answers!

Problem 2: Square subtraction. Consider the subtraction game with subtraction set the set of squares \(S = \{1, 4, 9, 16, \ldots \}\). Prove that there are infinitely many P-positions.

Problem 3. Consider a variation of the subtraction game with set \{1, 2, 3, 4\}. The new rule is that instead of taking chips from the pile, a player may instead return 1,2,3, or 4 previously taken chips to the pile.
(a) Prove that this game is not progressicely bounded.
(b) Prove that starting with 100 chips, player 2 can still force a win no matter what player 1 does.

Problem 4: Misere Empty and Divide. There are two boxes. Initially, one box contains \(m\) chips and the other contains \(n\) chips. Such a position is denoted by \((m, n)\), where \(m > 0\) and \(n > 0\). The two players alternate moving. A move consists of emptying one of the boxes, and dividing the contents of the other between the two boxes with at least one chip in each box. There is a unique terminal position, namely \((1,1)\). Consider Misere rules: last player to move loses. Show that the only P-positions are of the form

\[(3k - 1, 1), (1, 3k - 1), \text{ or } (3k - 1, 3l - 1),\]

where \(k, l > 0\) are arbitrary natural numbers.