Recall: Binomial coeff. \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \): #choices of k out of n.

Multinomial coeff. \( \binom{n}{k_1, k_2} = \frac{n!}{k_1! \cdots k_e!} \): #ways to split n items to groups of size k_1, \ldots, k_e.

\[
(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}
\]

\[
(x+y+z)^n = \sum_{\alpha + \beta + \gamma = n} \binom{n}{\alpha, \beta, \gamma} x^\alpha y^\beta z^\gamma
\]

\[
(x_1 + \cdots + x_\ell)^n = \sum_{K} \binom{n}{K_1, \ldots, K_\ell} x_1^{K_1} \cdots x_\ell^{K_\ell} \quad \text{(most general)}
\]

Qn: what is \( P(\text{product of 5 dice} = 8) \)

Ans: \( P(1, 3, 2, 2 \text{ in some order}) = \frac{\binom{5}{3}}{6^5} = \frac{10}{6^5} \)
\[ P(\{1,1,1,2,4\} \text{ in some order}) = \frac{\binom{5}{1,1,3}}{6^5} = \frac{20}{6^5} \]

\[ P(\text{prod.} = 8) = \frac{10}{6^5} + \frac{20}{6^5} = \frac{30}{6^5} \]

\[ \binom{n}{k} = \binom{n}{k, n-k} \]

\[ P(\text{5 sp. 4 cl. 2 d: 2 he}) \text{ in 13 cards} : \]
\[ = \frac{\binom{13}{5} \binom{13}{4} \binom{13}{2} \binom{13}{2}}{\binom{52}{13}} \leftarrow \text{which cards in each suit} \]

\[ \frac{\binom{13}{5} \binom{4}{2} \binom{2}{2}}{4^{13}} \leftarrow 13 \text{ suits} \]

\[ S = \{ \text{CDPHSSSHCSS}, \ldots \} \]
Election example

Candidate A gets 1422
B gets 1405

Afterwards we find 101 ineligible voters
If each vote equally likely to come from the illegal votes, what is \( P(\text{true outcome has A winning}) \)?

This is an urn problem: If \( \geq 59 \) of the 101 votes are for A, then result changes.

\[
P(k \text{ illegal votes for A}) = \frac{\binom{1422}{k} \binom{1405}{101-k}}{\binom{2827}{101}}
\]

\( k - (101-k) \leq 17 \Rightarrow k \leq 59 \)
\[ P(A \text{ still wins}) = \sum_{k=0}^{50} \frac{\binom{1422}{k} \binom{1405}{101-k}}{\binom{2827}{101}} \approx 0.94 \]

\[ E_m = \{ m \text{ of the 101 are A-votes} \} \]

Out of \( N \) people, how much is \( P(E) \), where \( E = \{ \text{two have same birthday} \} \)?

Q: \( P(4 \text{ different numbers on 4 dice}) = ? \)

\[ |E| = 6 \cdot 5 \cdot 4 \cdot 3 \quad |S| = 6^4 \quad \text{so} \quad P(E) = \frac{6 \cdot 5 \cdot 4 \cdot 3}{6^4} \]
Birthday problem & sol.: 

\[ |\mathcal{S}| = 365^N \]

\[ |E| = 365 \times 364 \times 363 \times \cdots \times (365-N) \]

\[ = \frac{365!}{(365-N)!} \]

So

\[ P(E^c) = \frac{365!}{(365-N)!} \]

\[ \frac{1}{365^N} \]

\[ P(E) = 1 - P(E^c) \]

\[ P(E) \approx e^{-(N/2)/365} \]