Events

Sample space: \( S = \text{set of possible outcomes.} \)

Events: \( E \) is a subset of \( S \).

e.g. \( S = \{H,T\} \) \( \phi = \{\} \), \( \{H\}, \{T\}, \{H,T\} = S \)

\( \text{events.} \)

e.g. \( S = \{1, 2, \ldots, 6\} \) there are \( 2^6 = 64 \) events

Complement: \( E^c \) (or \( \overline{E} \))

\( E^c = S \setminus E \) all outcomes not in \( E \).

\( S^c = \phi \) \( \phi^c = S \) \( (E^c)^c = E \) for any \( E \).
Intersection

$E \cap F$ or $EF$

$E \cap F = \text{all outcomes in both } E, F.$

e.g. $S = \{1, \ldots, 6\}$

$E = \text{odd, } F = \text{prime } \Rightarrow \{2, 3, 5\}$

$E \cap F = \{3, 5\}$

$E \cap (F \cap G) = (E \cap F) \cap G$  similarly for more.
Disjoint events: \( E, F \) are disjoint if \( E \cap F = \emptyset \)

def events \( E_1, E_2, E_3, \ldots \) are disjoint if any two \( E_i, E_j \) are disjoint.

Note: \( E \cap E^c = \emptyset \) for all \( E \).

Union:
\( E \cup F = \) all outcomes in \( E \) or \( F \) (or both)
\text{UFUG} : \text{outcomes in at least one of } E, F, G

\[ E \cup E^c = S \text{ for any } E. \]

\textbf{Properties}

\begin{align*}
\text{distributive laws} & \quad \circ \quad (E \cap F) \cup G = (E \cup G) \cap (F \cup G) \\
\circ \quad (E \cup F) \cap G = (E \cap G) \cup (F \cap G) \\
\end{align*}

\text{de Morgan:} \quad \text{complement of intersection is union of complements}

\[ (E \cap F)^c = E^c \cup F^c \]
Generally: \( (\bigcap_{i=1}^{k} E_i)^c = \bigcup_{i=1}^{k} E_i^c \)

Similarly: \( (\bigcup_{i=1}^{k} E_i)^c = \bigcap_{i=1}^{k} E_i^c \)

**Probability**

Assign each event a probability \( P(E) \) describing how often / what fraction of the time \( E \) occurs.

**Axioms of probability:**

1. For any \( E \), \( 0 \leq P(E) \leq 1 \).
2. \( P(\emptyset) = 0 \), \( P(S) = 1 \).
3. If \( E_1, E_2, \ldots \) are disjoint, then \( P(\bigcup E_i) = \sum_i P(E_i) \).
e.g. \( S = \{ (a, b) \mid 1 \leq a, b \leq 6 \} \) outcomes of two die rolls

\[ E = \{ a+b = 7 \} \quad F = \{ a+b = 18 \} = \{ (5,6), (6,5) \} \]

\[ P(E) = \frac{6}{36} \quad P(F) = \frac{2}{36} \]  
so  
\[ P(E \cup F) = \frac{6}{36} + \frac{2}{36} \]

**Example:** If \( S \) is a finite set, uniform probab. on \( S \) is defined by

\[ P(A) = \frac{|A|}{|S|} \]

\[ P(a=6 \text{ or } b=6) = \frac{11}{36} \]

\[ \neq P(a=6) + P(b=6) \]

\[ \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \]