Mathematics 101 — Solutions for week 7 quiz

Question 1 — 8 marks

Compute the following integrals

(a) $\displaystyle\int \tan^3\theta \sec\theta \, \mathrm{d}\theta$

(b) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{3+2x-x^2}}$

Question 2 — 4 marks

Write $\dfrac{7x-1}{(x-1)(2x+1)}$ in partial fraction form. Do not integrate it.

Solutions Q1 — 4 marks each

(a) Trig integral. Sub $u = \sec\theta$ so $\mathrm{d}u = \sec\theta\tan\theta \, \mathrm{d}\theta$

$$\begin{aligned}\int \tan^3\theta\sec\theta \, \mathrm{d}\theta &= \int \tan^2\theta(\tan\theta\sec\theta) \, \mathrm{d}\theta \\ &= \int (\sec^2\theta - 1) \, \mathrm{d}u \\ &= \int (u^2-1) \, \mathrm{d}u \\ &= \frac{1}{3}u^3 - u + c \\ &= \frac{1}{3}\sec^3\theta - \sec\theta + c\end{aligned}$$

(b) Trig substitution. Complete the square and then sub.

$$\begin{aligned}\int \frac{\mathrm{d}x}{\sqrt{3+2x-x^2}} &= \int \frac{\mathrm{d}x}{\sqrt{4-(x-1)^2}} \qquad (x-1) = 2\sin\theta \; \mathrm{d}x = 2\cos\theta \, \mathrm{d}\theta \\ &= \int \frac{2\cos\theta \, \mathrm{d}\theta}{\sqrt{4-4\sin^2\theta}} \\ &= \int \frac{2\cos\theta \, \mathrm{d}\theta}{\sqrt{4\cos^2\theta}} \\ &= \int \frac{2\cos\theta \, \mathrm{d}\theta}{2\cos\theta} = \int 1 \, \mathrm{d}\theta \\ &= \theta + c\end{aligned}$$

But $\sin\theta = (x-1)/2$, so

$$\int \frac{\mathrm{d}x}{\sqrt{3+2x-x^2}} = \sin^{-1}\left(\frac{x-1}{2}\right) + c$$

Solutions Q2 — 4 marks

Partial fractions question. Degrees are okay, so we split.

$$\begin{aligned}\frac{7x-1}{(x-1)(2x+1)} &= \frac{A}{x-1} + \frac{B}{2x+1} \\ &= \frac{A(2x+1)+B(x-1)}{(x-1)(2x+1)}\end{aligned}$$

Compare coefficients of x in the numerator gives

$$7 = 2A + B \qquad\qquad -1 = A - B$$

Add these to get $6 = 3A$. So $A = 2$ and $B = 3$. Hence

$$\frac{7x-1}{(x-1)(2x+1)} = \left(\frac{2}{x-1} + \frac{3}{2x+1}\right)$$