Mathematics 220

Practice Quiz 2 — 10 minutes

- The quiz consists of 2 pages and 2 questions worth a total of 6 marks.
- No work on this page will be marked.
- Fill in the information below before turning to the quiz.

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For each of the following quantified statements, give their negations. Then determine whether they are true or false. You must prove your answers — simply stating true or false is insufficient.

1. **3 marks** \( \forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} \text{ so that } (x \mid y) \iff (y \mid x). \)

**Solution:** Negation:

\[ \exists x \in \mathbb{Z} \text{ so that } \forall y \in \mathbb{Z}, (x \mid y) \text{ XOR } (y \mid x) \]

True.

Proof. Let \( x \) be any integer and then set \( y = x \). Hence \( x \mid y \) and \( y \mid x \) are both true statements and the biconditional is true. \( \square \)

2. **3 marks** \( \exists x \in \mathbb{Z} \text{ so that } \forall y \in \mathbb{Z}, (x \nmid y) \implies (y \mid x). \)

**Solution:** Negation:

\[ \forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} \text{ so that } (x \nmid y) \land (y \nmid x) \]

True.

Proof. Let \( x = 0 \) and let \( y \) be any integer. Then we can always write \( x = 0 = y \cdot 0 \), so the conclusion is always true. Since the conclusion is always true, the implication is true. \( \square \)

Alternatively:

Proof. Let \( x = 1 \) and let \( y \) be any integer, then since \( y = x \cdot y = 1 \cdot y \), it follows that \( x \mid y \). Since the hypothesis of the implication is false, the implication is true. \( \square \)