Mathematics 220 — Midterm — 45 minutes

October 17th 2018

- The quiz consists of 6 pages and 5 questions worth a total of 23 marks.

- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

- No work on this page will be marked.

- Fill in the information below before turning to the quiz.

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1. **5 marks** (a) Write the negation of the following statement:

“For every $n \in \mathbb{Z}$, $(n + 1)^2 \geq 9$ or $n < 2$.”

(b) Write the negation of the following statement:

“There exists $x \in \mathbb{Z}$ so that for every $r \in \mathbb{Q}$, if $x \neq 0$ then $rx = r$.”

(c) Write the converse and contrapositive of the following statement:

“ If you do not like chocolate, then you do not like brownies.”

Be sure to indicate which is which.

(d) Let $a \in \mathbb{N}$ with $a \geq 2$. Carefully define what “$a$ is prime” means.
2. 5 marks Let $n$ be an integer. Prove or disprove the following two statements

(a) If $3 \nmid n$, then $3 \mid (n^2 - 6n + 5)$.
(b) If $3 \nmid n^2$ then $3 \mid (n^2 + 2n)$. 
3. **3 marks** Prove the following statement

Let $x \in \mathbb{R}$. If $x > 0$ then \( \frac{2x}{x+1} \leq \frac{x+1}{2} \).
4. [4 marks] Prove the following statement.

For any integer \( n \geq 1, \)

\[
\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} \leq 2 - \frac{2}{(n+1)!}.
\]

Recall that \( n! = 1 \times 2 \times 3 \times \cdots \times n \)
5. 6 marks Let $S = \{n \in \mathbb{Z} : 3 \nmid n\}$.

Determine whether the following three statements are true or false — explain your answers ("true" or "false" is not sufficient).

(i) $\exists x \in S \text{ s.t. } \exists y \in S \text{ s.t. } x + y \in S$.

(ii) $\exists x \in S \text{ s.t. } \forall y \in S, x + y \in S$.

(iii) $\forall x \in S, \exists y \in S \text{ s.t. } x + y \in S$. 