Homework 2

The chapters refer to the course textbook, “The book of proof”.

(1) Chapter 4: Question 4
(2) Chapter 4: Question 6
(3) Chapter 4: Question 8
(4) Chapter 4: Question 18

(5) Prove the following:

   (a) Let $n$ be a nonzero integer. If $n^2 | n$, then $n = -1$ or $n = 1$.

   (b) If $n$ is an odd integer, then $4 | ((n - 1)(n + 1))$.

The next questions are a little harder and so we’ve given you some hints to help you on your way.

(6) Prove the following

Let $x \in \mathbb{R}$. If $x < 0$, then $x + \frac{1}{x} < -1$.

**Hint:** Try multiplying both sides of this inequality by $x$.

(7) Prove that

If $n \in \mathbb{N}$ is not prime, then $2^n - 1$ is not prime.

**Hint:** You might find the following fact helpful:

If $k \in \mathbb{N}$ then $(a^k - b^k) = (a - b)(a^{k-1} + a^{k-2}b + a^{k-3}b^2 + \cdots + ab^{k-2} + b^{k-1})$. 