

## STAGE 4 QUESTIONS, 2005 REGIONAL

1. A zoo has some ostriches and some tigers. Altogether they have 34 eyes and 48 legs. How many tigers are there? (Ostriches have 2 eyes and 2 legs, tigers 2 eyes and 4 legs.)
2. How many integers  $n$  satisfy the inequality  $(n - 6)(2n + 7) < 0$ ?
3. What is the surface area, in square centimetres, of the outside of a 50 cm by 50 cm by 40 cm closed cardboard box?
4. When the number  $N$  is divided by 8, the quotient is 17 and the remainder is 5. What is  $N$ ?
5. The rain started at 9:20 am, and continued at a steady rate all day. By 12:00 noon, 4 mm of rain had fallen. How many mm of rain fell from 7:20 am to 10:00 pm?
6. If  $x * y = x^2 + 4xy + y^2$ , what is  $45 * 5$ ?
7. Express  $\frac{2^{100}}{2^{101} - 2^{100} + 2^{99}}$  as a fraction in lowest terms.
8. How many real numbers satisfy the equation
$$(x)(x^2 - 1)(x^3 - 2)(x^4 - 3) = 0?$$
9. The *harmonic mean* of  $a$  and  $b$  is  $\frac{1}{\frac{1}{2} \left( \frac{1}{a} + \frac{1}{b} \right)}$ . What is the harmonic mean of 40 and 60?
10. What is the sum of the integers from 1 to 50 that are *not* divisible by 5?
11. Square  $\mathcal{A}$  has area 192 square centimetres, which is 75% of the area of square  $\mathcal{B}$ . How many centimetres are in the side of square  $\mathcal{B}$ ?

12. How many ordered pairs  $(x, y)$  are there such that  $x$  and  $y$  are integers and  $xy = 64$ ?

13. Joan completes a 26.2 mile race in 2.5 hours. What is her average speed in miles per hour? Give the answer correct to 2 decimal places.

14. A box of 50 cookies is divided between A, B, C, and D. Together, A and D get 24 cookies; A, B, and C together get 40 cookies. How many cookies does A get?

15. What is the product of 20.5 and 30.5, correct to 2 decimal places?

16. In how many ways can 6 (different) CDs be given to Adam, Beth, and Charlie so each gets 2 CDs?

17. What is the smallest prime that is the sum of 3 distinct primes?

18. If  $\frac{x+y}{x+2y} = \frac{7}{11}$ , what is  $\frac{x}{y}$ ?

19. A survey of 200 students showed that 150 like cupcakes, 120 like muffins, and 20 like neither. How many of the students like both cupcakes and muffins?

20. Two integers add up to 99. What is the largest possible value of their product?

21. Simplify:  $\frac{5^2 - 1}{3^2 - 1} \cdot \frac{9^2 - 1}{7^2 - 1} \cdot \frac{13^2 - 1}{11^2 - 1}$

22. What is the least positive integer that is divisible by all of  $1^2$ ,  $2^2$ ,  $3^2$ ,  $4^2$ ,  $5^2$ , and  $6^2$ ?

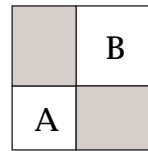
23. The average of two common fractions is  $1/3$ . If one fraction is  $1/5$ , what is the other fraction?

24. What is the largest prime factor of 2772?

25. The area of a triangle with vertices  $(-1, 0)$ ,  $(a, 0)$ , and  $(0, a)$  is 36. If  $a > -1$ , what is  $a$ ?

26. What is the smallest prime number that is greater than 199?

27. The areas of squares A and B in the picture are 25 and 36 square units respectively. How many square units are in the combined area of the two shaded rectangles?



28. How many pairs  $(x, y)$  of positive integers are there such that  $x + 2y = 50$ ?

29. Each of two dice has the numbers 1, 3, 5, 7, 9, and 11 on its faces instead of the usual 1, 2, ..., 6. If you toss the two dice, what is the probability of getting a sum of 10?

30. What is the greatest common factor of 144 and 264?

31. Let  $\mathcal{L}$  consist of all points in the coordinate plane that have integer coordinates. What is the sum of the  $x$ -coordinates of all points in  $\mathcal{L}$  that are at distance 5 from the point  $(10, 0)$ .

32. What is the largest integer that is less than 2005 and is divisible by 21?

33. Two different numbers are chosen at random from 1, 2, 3, ..., 10. What is the probability that one of the numbers is twice the other? Express the answer as a fraction in lowest terms.

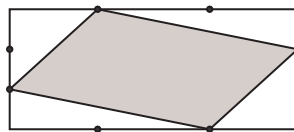
34. The point  $P$  is on the line segment that joins  $(2, 8)$  and  $(3, 27)$ , and halfway between them. What is the positive difference of the coordinates of  $P$ ?

35. The side lengths of a triangle are 10 units, 13 units, and 13 units. How many square units are in the area of the triangle?

36. What is the largest prime factor of  $24^4 - 24^2$ ?

37. What is  $99^2 - 1$ ?

38. Each side of a rectangle is divided into 3 equal parts. Some of the division points are joined as in the picture. If the rectangle has area 108 square metres, how many square metres are in the shaded region?



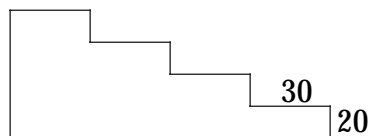
39. What is 112.5% of 56?

40. What is the least integer whose cube is greater than 10,000?

41. What is the greatest integer  $n$  such that  $3^n < 2^{n+3}$ ?

42. How many integers  $n$  satisfy the inequality  $|n| < 2005$ ?

43. How many square centimetres are in the area of the “staircase” figure below, given that each “stair” is 30 cm wide and 20 cm high?



44. What is the least perfect square that is greater than  $7^4$ ?

45. The sum of the integers from 1 to 100 is 5050. What is the sum of the integers from 201 to 299?

46. Beth left the campground at 8:45 in the morning, and arrived home at 11:05 the same morning. The distance from campground to home is 189 km. What was Beth’s average speed, in km per hour?

47. What is the units digit of  $1^2 + 2^2 + 3^2 + \dots + 30^2$ ?

48. Simplify:  $\left(10 + \frac{3}{10}\right)^2 - \left(10 - \frac{3}{10}\right)^2$ .

49. Let  $\ell$  be the line that goes through the points  $(0, 2)$  and  $(3, 11)$ . What is the  $x$ -coordinate of the point where the line  $\ell$  meets the  $x$ -axis?

50. If  $16y + 5 = 9y - 1$ , what is the value of  $28y$ ?

51. Amy has a total of \$16.00 in nickels, dimes, and quarters. If Amy has an equal number of coins of each kind, what is the total number of coins that Amy has?

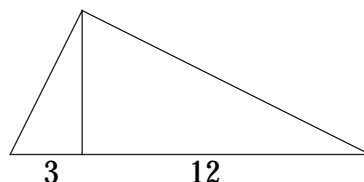
52. A movie ended at 11:00 pm and ran for 108 minutes. At what time did the movie start?

53. Solve for  $x$ :  $\frac{1}{15} - \frac{1}{18} = \frac{1}{x}$ .

54. A 48 centimetre-long piece of wire is cut into two parts, one twice as long as the other. Each part is bent to form a square. What is the sum, in square centimetres, of the areas of the two squares?

55. What is the remainder when 200520052005 is divided by 9?

56. The altitude to the hypotenuse of a right-angled triangle divides the hypotenuse into segments of length 3 and 12 units. How many square units are in the area of the triangle?



57. Solve for  $x$ :  $|x + 6| = |x - 66|$ .

58. Alan bought three textbooks, and spent a total of \$210. The calculus book cost \$5 more than the economics book, which cost \$5 more than the psychology book. How many dollars did the calculus book cost?

59. How many whole numbers between 1 and 100 (inclusive) are divisible by neither 2 nor 5?

60. What is the greatest integer  $n$  such that  $99!$  is divisible by  $99^n$ ?

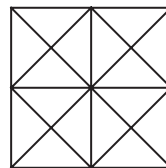
61. Alphonse spent one-third of his money, and then lost one third of what remained. After that, he had \$20 left. How many dollars did he start with?

62. You toss two fair standard dice, a red one and a blue one. What is the probability that the number on the red one is bigger than the number on the blue one? Express your answer as a fraction in lowest terms.

63. Six is five percent of four percent of what number?

64. Given that  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  are five consecutive integers with  $b + c + d = 135$ , what is  $a + b + c + d + e$ ?

65. How many triangles (including triangles of all sizes) are in the picture below?



66. If  $f(x) = x^2 - 2x + 2$ , what is  $f(f(f(3)))$ ?

67. A litre of regular gas costs \$0.70. A litre of premium costs \$0.80. Richie spent \$36 to fill up with premium. How much could he have saved by filling up with regular? Give your answer in dollars and cents.

68. A circle passes through the points  $(0, 0)$ ,  $(10, 0)$ , and  $(0, 10)$ . Find the area of the circle. Express your answer in terms of  $\pi$ .

**69.** There are 120 five-digit numbers that can be formed using each of the digits 1, 2, 3, 4, and 5 once. How many of these are divisible by 4?

**70.** Three distinct circles and one line are drawn in the plane. What is the largest possible number of points where two or more of these figures meet?

**71.** A fair coin is flipped 4 times in a row. What is the probability that we get 3 or more consecutive heads during these 4 tosses? Express your answer as a fraction in lowest terms.

**72.** Express  $x$  as a common fraction, given that  $1 - \frac{1}{1 - \frac{1}{x}} = 7$

**73.** Simplify:  $1 - 2 + 3 - 4 + 5 - 6 + \dots + 99 - 100$ .

**74.** A movie started at 7:47 pm and ended at 9:37 pm. At what time was the movie exactly halfway through?

**75.** How many perfect squares are there among the positive integers that are less than 10000 and are multiples of 99?

**76.** What is the 2005<sup>th</sup> term of the arithmetic sequence 1, 6, 11, 16, ...?

**77.** How many more square metres are there in the area of a circle with diameter 20 than in the area of a circle with diameter 10? Express your answer in terms of  $\pi$ .

**78.** Two fair dice are tossed. What is the probability that the product of the numbers showing is a prime number? Express your answer as a fraction in lowest terms.

**79.** What is the leftmost digit in the decimal representation of  $9^{100} + 10^{100}$ ?

**80.** Express  $\frac{8! - 7!}{8! + 7!}$  as a fraction in lowest terms.

**81.** Given that 5 is a solution of the equation  $x^2 + ax + 5a = 2005$ , what is the value of  $a$ ?