Primitive and doubly primitive divisors in dynamical sequences by Khoa D. Nguyen (UBC and PIMS)

Let $K$ be a number field or a function field of characteristic 0, let $\varphi(z) \in K(z)$ having degree at least 2 and let $\alpha \in K$ such that the orbit $\{\varphi^n(\alpha)\}_{n \geq 0}$ is infinite. Consider the question: (A) except trivial counter-examples, is it true that for all sufficiently large $n$, the element $\varphi^n(\alpha)$ has a prime divisor $p$ that is not a divisor of $\varphi^k(\alpha)$ for every $k < n$. Ingram and Silverman are the first to consider this question in such generality. They even go further and ask: (B) except trivial counter-examples, is it true that for all $m \geq 0$ and $n > 0$ such that $m + n$ is sufficiently large, the element $\varphi^{m+n}(\alpha) - \varphi^m(\alpha)$ has a prime divisor $p$ that is not a divisor of any $\varphi^{M+N}(\alpha) - \varphi^M(\alpha)$ for $M < n$ or $N < n$. Later on, Faber and Granville modify question (B) somewhat and provide certain evidence towards it.

In this talk, we explain how the $ABC$ Conjecture implies that both questions have an affirmative answer. In the function field case our result is unconditional; when using a deep result of Yamanoi (previously conjectured by Vojta), we can show that $p$ appears with multiplicity 1. This is joint work with Chad Gratton and Tom Tucker for Question (A), and with Dragos Ghioca and Tom Tucker for Question (B).