ROTH'S THEOREM IN THE PRIMES

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ABSTRACT. In 1939, Van Der Corput proved that the set of primes $\mathcal{P}$ contains infinitely many non trivial three term arithmetic progressions. In 2003, Green proved an anologue of Roth's theorem, and showed that any subset $A \subset \mathcal{P}$ with positive relative density must contain infinitely many three term arithmetic progressions. Suppose that $A \subset \{1, \ldots, N\}$ is a set of primes containing, and let $\alpha = |A|/\pi(N)$ be the relative density of A in $\{1, \ldots, N\}$, where $\pi(N)$ denotes the number of primes in the set $\{1, \ldots, N\}$. Helfgott and Roton improved Green's quantitative result, proving A must contain a three term arithmetic progression when

$$\alpha \gg \frac{\log \log \log N}{(\log \log N)^{1/3}},$$

We improve their density bound, showing that if there exists $\epsilon > 0$ such that

$$\alpha \gg_\epsilon \frac{1}{(\log \log N)^{1-\epsilon}},$$

then A contains a three term arithmetic progression.