A Discontinuous Galerkin Method for Vlasov-Like Systems

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This talk will be an amalgamation of aspects of scientific computing – development, verification, and interpretation – with application to the Vlasov-Poisson (VP) system, an important nonlinear partial differential equation containing the essential difficulties of collisionless kinetic theories. I will describe our development of a discontinuous Galerkin (DG) algorithm,[1,2] its verification via convergence studies and comparison to known Vlasov results, and our interpretation of computational results in terms of dynamical systems ideas.

The DG method was invented for solving a neutron transport model, successfully adapted to fluid motion including shock propagation, applied to the Boltzmann equation, and developed in the general context of conservation laws, and elliptic and parabolic equations. Our development for the VP system required the simultaneous approximation of the hyperbolic Vlasov equation with the elliptic Poisson equation, which created new challenges.

I will briefly discuss advantages of the method and describe our error estimates and recurrence calculations for polynomial bases. Then, I will show results from a collection of benchmark computations of electron plasma dynamics, including i) convergence studies of high resolution linear and nonlinear Landau damping with a comparison to theoretical parameter dependencies ii) the nonlinear two-stream instability integrated out to (weak) saturation into an apparently stable equilibrium (BGK) state with detailed modeling of this state, and iii) an electric field driven (dynamically accessible) example that appears to saturate into various periodic solutions. I will interpret such final states, in analogy to finite-dimensional Hamiltonian theory, as Moser-Weinstein periodic orbits, and suggest a possible variational path for proof of their existence.

Finally, I will comment briefly on recent progress on extensions to the Maxwell-Vlasov system, including estimates and computational results.[3]

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[1] R. E. Heath, Ph.D. Thesis, University of Texas (2007); R. E. Heath, I. M. Gamba, P. J. Morrison, and C. Michler, "A discontinuous Galerkin method for the Vlasov-Poisson system, J. Comp. Phys. **231**, 1140 (2012)

[2] Y. Cheng, I. M. Gamba, P. J. Morrison, "On Runge–Kutta discontinuous Galerkin schemes for the Vlasov–Poisson systems", preprint (2012)

[3] Y. Cheng, F. Li, I. M. Gamba, P. J. Morrison, J.-M. Qiu, and G. Lin, "A discontinuous Galerkin scheme for the Vlasov-Maxwell system: application to the Weibel instability," in preparation.