

Let $A = (a_0, a_1, \dots, a_{n-1})$ be a finite sequence of complex numbers with modulus 1 of length n . Let

$$A(z) := a_0 + a_1 z + \dots + a_{n-1} z^{n-1}$$

be the unimodular polynomial associated with the sequence A and $z := e^{2\pi i/n}$. In this talk, we give an exact formula of the L_4 norm for A over the unit circle, namely, if n is an odd positive integer, then

$$\begin{aligned} \|A\|_4^4 = & \frac{1}{n} \sum_{a=0}^{n-1} |A(\zeta^a)|^4 - \frac{4}{n^3} \sum_{a=0}^{n-1} |A(\zeta^a)|^2 \Re \left(A(\zeta^a) \overline{A_2(\zeta^a)} \right) \\ & - \frac{4}{n^3} \sum_{a=0}^{n-1} \Re \left(A(\zeta^a)^2 \overline{A_1(\zeta^a)^2} \right) + \frac{8}{n^3} \sum_{a=0}^{n-1} |A(\zeta^a) A_1(\zeta^a)|^2, \end{aligned}$$

where

$$A_1(z) := \sum_{\ell=0}^{n-1} \ell a_\ell z^\ell \quad \text{and} \quad A_2(z) := \sum_{\ell=0}^{n-1} \ell^2 a_\ell z^\ell.$$

Using this formula, we are able to prove that if $\overline{a_{n-\ell}} = \varepsilon a_\ell$ for all $1 \leq \ell < n$ for some fixed complex number ε with $|\varepsilon| = 1$ and $a_0 := \varepsilon^{-1/2}$, then we have

$$\|A\|_4^4 \geq \frac{5}{3} n^2 - 2n + \frac{4}{3}.$$

The main term of lower bound is the best possible and is attained by $A(z) := 1 + \sum_{a=1}^{p-1} \left(\frac{a}{p} \right) z^a$, where $\left(\frac{\cdot}{p} \right)$ is the Legendre symbol and p is a prime $\equiv 1 \pmod{4}$. As a corollary, we can show that if $A(z)$ is a reciprocal polynomial of even degree $n-1$, then $\|A\|_4^4 \geq \frac{5}{3} n^2 + O(n^{3/2})$. Also, our result shows that the largest asymptotic merit factor for reciprocal Littlewood polynomials of even degree is $3/2$.