Abstract: Define $e_n(t) = \{t/n\}$. Let d_N denote the distance in $L^2(0, \infty; t^{-2}dt)$ between the indicator function of $[1, \infty[$ and the vector space generated by $e_1, \dots, e_N$. A theorem of Báez-Duarte (2003) states that the Riemann hypothesis (RH) holds if and only if $d_N \to 0$ when $N \to \infty$. Assuming RH, we prove the estimate

$$d_N^2 \le (\log \log N)^{5/2+o(1)} (\log N)^{-1/2}.$$

I shall put this result in its historical context, from Nyman's criterion (1950) and its beautiful proof to a sketch of our proof. I shall focus on the main ingredient we used, a method of Maier and Montgomery, recently sharpened by Soundararajan, to get some upper bound for partial sums of the Möbius function. (joint work with Michel Balazard)