NONUNIQUENESS FOR A PARABOLIC SPDE WITH $\frac{3}{4}-\varepsilon$-HÖLDER DIFFUSION COEFFICIENTS

EDWIN PERKINS

We prove an analogue of the Girsanov examples for SDE's for the parabolic stochastic partial differential equation (SPDE)

$$\frac{\partial X}{\partial t} = \frac{\Delta}{2} X(t,x) + |X(t,x)|^p \dot{W}(t,x),$$

with zero initial conditions. Here $\dot{W}$ is a space-time white noise on $R_+ \times R$. More precisely, we show the above stochastic pde has a non-zero solution for $0 < p < 3/4$, and hence solutions are not unique in law or pathwise unique. The case $p = 1/2$ arises as a scaling limit point of a system of branching annihilating random walks. This shows that an SPDE analogue of Yamada-Watanabe's well-known theorem for pathwise uniqueness of SDE's (Mytnik and Perkins, PTRF 2011) is essentially sharp. This is joint work with Carl Mueller and Leonid Mytnik.