Prime numbers are central objects of study in number theory. In the 1730s, Euler gave a novel proof of the infinitude of primes by showing that $\sum p \frac{1}{p}$ diverges, where the sum runs over all prime numbers. Deriving inspiration from Euler’s idea, Dirichlet proved the infinitude of primes in arithmetic progressions in 1837. His proof relied on the fact that $\sum_{n=1}^{\infty} \frac{\chi(n)}{n} \neq 0$, where $\chi$ is a periodic multiplicative function taking values on the unit circle. Intrigued by this curious non-vanishing result, in the early 1960s, S. Chowla initiated a study of values of the $L$-function, $L(s, f) := \sum_{n=1}^{\infty} \frac{f(n)}{n^s}$, for any periodic function $f$ on $\mathbb{Z}$, at positive integer arguments. In this talk, we will discuss how methods from analytic, algebraic and transcendental number theory come together harmoniously, giving rise to a beautiful theory of these special values. Around the same time as Chowla, Erdős conjectured that the series $\sum_{n=1}^{\infty} \frac{f(n)}{n} \neq 0$ whenever it converges, for certain periodic functions $f$, taking values in $\{-1, 0, 1\}$. This conjecture was proved by M. R. Murty and N. Saradha when $q$, the period of $f$, satisfies $q \equiv 3 \text{ mod } 4$. This conjecture remains open in the case $q \equiv 1 \text{ mod } 4$. Using probabilistic techniques, we show that Erdős’s conjecture holds with ‘probability’ one.