Mathematics 601 (Topics in Analysis)

Harmonic analysis and the geometry of fractals

Winter/Spring 2015

Instructor: I. Laba (Math Bldg 200, 604-822-4457, ilaba(at)math.ubc.ca).
Lectures: Tue Th 11-12:30, MATX 1118.
Office hours: TBA.

This course will draw on, and connect, topics from harmonic analysis, geometric measure theory, and additive combinatorics. Singular and oscillatory integrals associated with submanifolds of Euclidean spaces, and their relation to the underlying geometry, have long been a mainstay of Euclidean harmonic analysis. We will take a look at extending this line of research to the measure-theoretic setting of fractal sets, where the geometric ideas from classical harmonic analysis have arithmetic analogues inspired by additive combinatorics. Tentatively, the topics will include the following:

- Hausdorff and Fourier dimension
  - Hausdorff dimension and energy integrals
  - Fourier dimension and Salem sets
  - Applications to geometric measure theory: convolutions, projections, distances
- Maximal functions and differentiation theorems
  - The Hardy-Littlewood maximal function
  - The spherical maximal function
  - Maximal functions and differentiation theorems for fractal sets
- Restriction estimates
  - The Tomas-Stein theorem
  - Restriction theorems for fractals

My ICM paper has a quick preview of some of these topics, Here, we will do it more systematically and in depth. The course should provide a good foundation for anyone who might be interested in research in this area.

Your course grade will be based on an in-class presentation on a topic related to the course material. You will also have to prepare a written summary to be handed out in class. Please discuss the choice of presentation topic with me in advance, no later than February 13 (last day before the winter break). If there is a particular topic that you would like to present feel free to suggest it.

Resources: (this list will be updated as we go)

- *Fourier Analysis*, J. Duoandikoetxea, American Mathematical Society, 2001
- *Geometry of sets and measures in Euclidean spaces: fractals and rectifiability*, P. Mattila, Cambridge Univ. Press, 2003
- *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, E.M. Stein, Princeton Univ. Press, 1993
- *Real Analysis*, E.M. Stein and R. Shakarchi, Princeton Univ. Press, 2003

- *Lectures on Harmonic Analysis*, T. Wolff, American Mathematical Society, 2003. I have ordered it as a "recommended textbook" so you should be able to get it from the bookstore, but you can also download the PDF from my webpage.

Prerequisites: MATH 541, or equivalent background in basic harmonic analysis.