



# Mathematics Department

- [Course Outline](#)
- [Assignments](#)
- [Project](#)
- [Grading](#)
- [Resources](#)

## MATH 564: Evolutionary Dynamics

**Meeting Times:** Tuesday, Thursday, 11:00 - 12:30

**First Class:** Thursday, September 5<sup>th</sup>, 2013

**Location:** MATX 1118

### Instructor

[Christoph Hauert](#)

**Office:** Mathematics, Room 234

**Hours:** by appointment

**Email:** [hauert@math.ubc.ca](mailto:hauert@math.ubc.ca) (please indicate 'math 564' in subject line)

### Announcements

- None

### Course Outline

Evolution is the unifying theme in biology. Evolutionary processes are responsible for the emergence of the rich variety of species across the planet. Cooperation represents one of the key organizing principles in evolution, and the history of life and of societies could not have unfolded without the repeated cooperative integration of lower level units into higher level entities. Evolutionary theories have attracted increasing attention from other behavioral disciplines including sociology and economics. This has led to the notion of cultural evolution aiming at a better understanding of human cooperation including the emergence of social norms. Cultural evolution follows the same basic selection principle as biological evolution but the lack of the genetic constraints of mutation, recombination and inheritance results in a largely unexplored dynamics governed by the more flexible mechanisms of innovation, learning and imitation.

## Goals

This course provides a sound introduction into mathematical models of evolution and the theory of games. Modeling techniques that are covered include: stochastic dynamics of invasion and fixation of mutants in a finite population; evolutionary game theory and frequency dependent selection -- each agents' performance is affected by everyone else; adaptive dynamics and the process of diversification and speciation through evolutionary branching; modeling spatially structured populations. In all cases the link to current challenges in research is emphasized by student presentations and discussions of the literature as well as by identifying potential research questions. Each student develops his/her own small research project in consultation with the instructor. At the end of the term, all students hand in a written report, present their project to the class and participate in a peer review process assessing the projects of their fellow students.

## Tentative Timeline

| Week                | Lecture Topic                                            | Notes                        |
|---------------------|----------------------------------------------------------|------------------------------|
| Week 1,<br>Sept. 5  | Introduction                                             |                              |
| Week 2,<br>Sept. 10 | Finite populations, constant selection                   |                              |
| Week 3,<br>Sept. 17 | Structured populations                                   |                              |
| Week 4,<br>Sept. 24 | Game theory, cooperation, classical analysis             | Discussions of presentations |
| Week 5,<br>Oct. 1   | Finite populations, frequency dependent selection        |                              |
| Week 6,<br>Oct. 8   | From finite to infinite populations, replicator dynamics | Presentations                |
| Week 7,<br>Oct. 15  | Structured populations, pair approximation               | Discussions of projects      |
| Week 8,<br>Oct. 22  | Cooperation, Reward, Punishment & Reputation             |                              |
| Week 9,<br>Oct. 29  | Ecological dynamics & evolutionary games                 |                              |
| Week 10,<br>Nov. 5  | Mutant Games                                             |                              |
| Week 11,<br>Nov. 12 | Continuous games, adaptive dynamics                      |                              |
| Week 12,<br>Nov. 19 | Origin of Cooperation                                    |                              |
| Week 13,<br>Nov. 26 | Project presentations                                    |                              |

## Prerequisites

This course combines various topics covered in undergraduate mathematics courses - including differential equations, dynamical systems, stochastic processes, probability, Markov chains, etc. However, committed graduate students from other disciplines that are willing to catch up on mathematical theories they might not be familiar with are encouraged to attend and stimulate discussions with problems from their fields. Knowledge of computer programming and mathematics software such as Maple, Mathematica or MATLAB might be helpful for the project work but are not required.

## Assignments

The homework assignments will be posted below. Late homework is not accepted.

- TBA

## Presentations & Project

Some suggestions for presentations and projects based on recent research results. For some of these topics ideas for manageable projects exist. If you are interested please contact me for more specific information. However, you are free (and encouraged) to choose any other research paper that catches your interest.

- **Evolutionary Dynamics - general**

*Literature:*

1. Nowak MA, A Sasaki, C Taylor, D Fudenberg (2004). *Emergence of cooperation and evolutionary stability in finite populations* Nature **428**: 646-650 ([PDF](#)).
2. Traulsen, A., Claussen J. C. & Hauert, Ch. (2005) *Evolutionary dynamics: from finite to infinite populations* Phys. Rev. Lett. **95**, 238701 ([PDF](#)).
3. Traulsen, A., Hauert, C., De Silva, H., Nowak, MA & Sigmund, K. (2009) *Exploration dynamics in evolutionary games* Proc. Natl. Acad. Sci. USA **106** 709-712 ([PDF](#)).
4. Traulsen, A., Claussen J. C. & Hauert, Ch. (2012) Stochastic differential equations for evolutionary dynamics with demographic noise and mutations Phys. Rev. E **85**, 041901 ([PDF](#)).

- **Cooperation**

*Literature:*

1. Sigmund, K., Hauert, C. & Nowak, M. (2001) *Reward & Punishment*, Proc. Natl. Acad. Sci. USA **98**, 10757-10762 ([PDF](#)).
2. Hauert, C, Traulsen, A., Brandt, H., Nowak, M. A. & Sigmund, K. (2007) *Via freedom to coercion: the emergence of costly punishment*, Science **316**, 1905-1907 ([PDF](#)).
3. Sigmund, K., Brandt, H., Traulsen, A., & Hauert, C. (2010) *Social learning promotes institutions for governing the commons*, Nature **466**, 861-863 ([PDF](#)).

4. Hilbe, C., Nowak, M.A. & Sigmund, K. (2013) *Evolution of extortion in Iterated Prisoner's Dilemma games* Proc Natl. Acad. Sci. USA **110** 6913-6918 ([PDF](#)).
- **Dynamics of spatial games**  
Literature:
    1. Ohtsuki, H., Hauert, C., Lieberman, E. & Nowak, M. (2001) *A simple rule for the evolution of cooperation*, Nature **441**, 502-505 ([PDF](#)).
    2. Hauert, C. & Doebeli, M. (2004) *Spatial structure often inhibits the evolution of cooperation in the Snowdrift game*, Nature **428**, 643-646 ([PDF](#)).
  - **Ecological Public Goods Games**  
Literature:
    1. Wakano, J., Nowak, M. & Hauert, C. (2009) *Spatial Dynamics of Ecological Public Goods*, Proc. Natl. Acad. Sci. USA **106**, 7910-7914 ([PDF](#)).
    2. Hauert, C., Wakano, J. & Doebeli, M. (2008) *Ecological Public Goods Games: cooperation and bifurcation*, Theor. Pop. Biol. **73**, 257-263 ([PDF](#)).
  - **Diversification**  
Literature:
    1. Doebeli, M., *Adaptive Diversification*, Princeton University Press, 2011.
    2. Doebeli, M., Hauert, C. & Killingback, T. (2004) *The evolutionary origin of cooperators and defectors*, Science **306**, 859-862. ([PDF](#))

## Project timeline

- TBA

## Project guidelines

The project paper should be a short, concise summary of your mini-research project with at most 12 pages (12pt font size, double spaced, excluding title page and references). As a target audience it might be best to think of your peers in class -- do not expect familiarity with the specifics of your project but you can rely on mathematical knowledge and some exposure to dynamical systems and evolutionary game theory. The paper should start with a brief introduction (~2 pages) that sketches the problem and puts it in a wider context and concludes with a discussion (~2 pages) that highlights the results and relates them to the broader context stated in the beginning. The model and results must be the central piece and should be described with sufficient detail that the reader can easily follow your line of argument but there is no need to show every step e.g. in your mathematical derivations. The emphasis must lie on an clear, intuitive and consistent presentation.

If necessary, a supplement may be enclosed -- for example containing detailed calculations, proofs and/or simulation details but the main text must be understandable without consulting the supplement.

## Peer review guidelines

For the peer review you have to write a brief report on the content, presentation and originality of the paper (max. 1 page). The reviewers remain anonymous but the content will be returned to the

author - the same as in real life (for some journals the reviews are double-blind but that is unlikely to work in such a small group). Just as some guidance, after or while reading the paper as yourself questions like: Is the problem well motivated? Is it an interesting and relevant problem? Is the model suitable to address the problem? Is the model well and convincingly presented? Is the derivation of the results clear? Do the conclusions follow from the model? Was it an interesting read? etc.

## Grading guidelines

Your grade for the course will be computed roughly as follows:

**Assignments:** (20%) 3-4 problem sets on material discussed in class.

**Presentation:** (15%) presentation of research article to the class.

**Term Project, Paper:** (30%) small research project.

**Term Project, Presentation:** (20%) presentation of research project to the class.

**Term Project, Peer Review:** (10%) review the term project papers of your peers.

**Participation:** (5%) contributions to discussions in class.

## Useful Resources

- Martin Nowak, *Evolutionary Dynamics*, Belknap Press, 2006.
- Karl Sigmund, *The Calculus of Selfishness*, Princeton University Press, 2010.
- Josef Hofbauer & Karl Sigmund, *Evolutionary Games and Population Dynamics*, Cambridge University Press, 1998.
- Nicholas Christakis & James Fowler, *Connected*, Little, Brown & Co., 2009.
- John Maynard Smith & Eörs Szathmáry, *The Major Transitions in Evolution*, W. H. Freeman & Co., 1995.
- [EvoLudo](#) and [VirtualLabs](#): Collections of interactive tutorials on the fascinating dynamical world of evolutionary processes.

Course webpage: [Course schedule](#).

[BACK TO TOP](#)

design by [Clive Goodinson](#) and adapted by Christoph Hauert.