MATH 555: Sparse Approximations and Compressed Sensing

Instructor: Ozgur Yilmaz

Time: Tuesday-Friday, 12:30-2pm

Course Material and Topics:

Recently, a new signal processing paradigm based on "sparsity" has emerged. This new paradigm exploits an empirical observation: many types of signals, e.g., audio, natural images, and video, can be well represented by a sparse expansion in terms of a suitable basis or frame, that is, by only a small number of significant basis elements. *Compressed sensing*, which has gained enormous interest in applied mathematics as well as in engineering in recent years, exploits this fact in order to recover signals from a very limited (what was previously considered highly incomplete) amount of linear measurements. Applications include signal acquisition, A/D conversion, radar, astronomy, seismology, and more.

Current rigorous results are mostly on recovery guarantees for certain random measurement ensembles. Consequently the mathematical theory of compressed sensing uses tools from non-asymptotic random matrix theory and geometry of Banach spaces.

Another important ingredient of the sparse approximation theory is related to algorithms: For sparse approximations and compressed sensing, at first sight, one needs to solve optimization problems that are of combinatorial nature. Surprisingly, under certain technical conditions, these combinatorial problems can be provably replaced by efficient recovery algorithms such as convex relaxation techniques leading to ℓ^1-minimization.

This course will introduce both mathematical and algorithmic aspects of sparse approximations and compressed sensing. A detailed outline is as follows:

- General overview
  - Sparse approximation and compressed sensing
  - Mathematical framework
  - Examples from applications: single pixel camera, sampling theory, compression, denoising, low-rank matrix recovery
- Sparse solutions of underdetermined systems
  - ℓ_0 minimization: sufficient conditions for recovery of sparse signals, stability and robustness issues, computational complexity
  - Tractable algorithms for sparse recovery: ℓ_1 minimization (basis pursit), OMP (orthogonal matching pursuit), thresholding-based methods
- Theoretical recovery guarantees for ℓ_1 minimization: deterministic analysis
  - Null space property
  - Coherence
  - Restricted isometry property (RIP)
- Random matrices and sparse recovery
  - Sub-Gaussian matrices and RIP
  - Non-uniform recovery
- Advanced topics

- Instance optimality and quotient property
- Non-convex optimization for sparse recovery
- Random sampling of bounded orthonormal systems
- Quantization theory for compressed sensing

Learning Objectives: The main learning objectives of this course are as follows: (i) The students will gain a broad perspective in the modern mathematical signal processing theory: the course will focus on the interplay between mathematical theory, algorithmic issues, and real-world applications. (ii) The students will acquire basic mathematical skills necessary to follow the literature in this field as well as the computational skills (e.g., using specialized toolboxes developed in Matlab) necessary to work on problems in a related area. (iii) The students will be exposed to several open research problems on a field that is extremely new and active. (Note that most of the results discussed in this course are less than 10 years old.)

Prerequisites: The course will be mostly self-contained. It would be beneficial to have some background in functional analysis and harmonic analysis and/or in signal processing and information theory.

Evaluation:

The instructional format for the course will consist of lectures of 3 hours per week.

50% of the mark is based on biweekly homework assignments, 50% on a final project and presentation.

Textbooks and References:

"A Mathematical Introduction to Compressive Sensing", Simon Foucart and Holger Rauhut. **Available online:**

`http://link.springer.com/book/10.1007/978-0-8176-4948-7/page/1`

Papers: (foundations of compressed sensing)

Emmanuel Candès, Justin Romberg, and Terence Tao, *Stable signal recovery from incomplete and inaccurate measurements*. (Communications on Pure and Applied Mathematics, 59(8), pp. 1207-1223, August 2006)

David Donoho, *Compressed sensing*. (IEEE Trans. on Information Theory, 52(4), pp. 1289 - 1306, April 2006)

Richard Baraniuk, Mark Davenport, Ronald DeVore, and Michael Wakin, *A simple proof of the restricted isometry property for random matrices*. (Constructive Approximation, 28(3), pp. 253-263, December 2008)

Albert Cohen, Wolfgang Dahmen, and Ronald DeVore, *Compressed sensing and best k-term approximation*. (J. Amer. Math. Soc. 22 (2009), pp. 211-231)

More resources: For more references on compressed sensing, see the Compressed Sensing Resources at: http://dsp.rice.edu/cs