Math 554 Symmetries and Differential Equations T2 2013/14

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The scheduled time for the course has been changed to Wed: 3-6pm in Math 103. Note that this is likely the last time this course will be offered at UBC.

Outline

This applied mathematics course is about symmetry methods (group methods) for solving ordinary and partial differential equations.

- Symmetry methods are algorithmic and hence amenable to symbolic computation.
- Symmetry methods systematically unify and extend well-known ad-hoc techniques, learned in undergraduate DE courses, to construct explicit solutions for ordinary and partial differential equations, especially for nonlinear DEs.
- It is essential to learn symmetry methods to use, understand and extend existing symbolic manipulation software for obtaining analytical results.

Topics

1. Dimensional analysis, modelling and invariance

- Buckingham Pi-Theorem
- applications to boundary value problems for partial differential equations
- generalization to invariance of boundary value problems under scalings of variables

2. Lie groups of point transformations and infinitesimal transformations

- extended transformations
- multi-parameter Lie groups of point transformations; Lie algebras
- mappings of curves and surfaces
- local transformations

3. Ordinary differential equations

- how to find systematically the Lie group of point transformations (point symmetries) admitted by a given ODE
- how to systematically reduce the order of an ODE from an admitted point symmetry
- how to systematically find integrating factors (conservation laws) and consequent first integrals for a given system of ODEs
- fundamental connections between integrating factors and symmetries
- applications to boundary value problems
- invariant solutions--separatrices and envelopes

4. Partial differential equations

- how to find systematically the Lie group of point transformations (point symmetries) admitted by a given PDE system
- how to construct systematically invariant (similarity) solutions from admitted point symmetries
- applications to boundary value problems

5. Miscellaneous topics in PDEs depending on interests of students e.g.,

- how to find systematically the conservation laws of a given PDE system
- how to determine systematically whether a nonlinear PDE system can be mapped invertibly to a linear PDE system and find such a mapping when one exists
- how to determine systematically whether a linear PDE with variable coefficients can be mapped invertibly to a linear PDE with constant coefficients and find such a mapping when one exists
- nonlocal symmetries
- nonclassical method for finding solutions of nonlinear PDEs.

The interests of students taking the course will determine the time spent on topics 3 and 4. If students are only interested in PDEs, then topic 3 will be omitted and most of the miscellaneous topics will be covered.

Pre-requisites: Elementary courses in differential equations and linear algebra. No knowledge of group theory will be assumed.

Course graded by assignments + project (no final exam)

Texts (available online at no charge through the UBC Library)

1. Bluman and Anco: *Symmetry and Integration Methods for Differential Equations.* Springer Appl. Math. Sci. Vol 154 (2002). This book is also published in China (in English, 2004; in Chinese, 2009).
2. Bluman, Cheviakov and Anco: *Applications of Symmetry Methods to Partial Differential Equations.* Springer Appl. Math. Sci. Vol 168 (2010). This book will be published in China (in English, 2014)