MATHEMATICS 401 2013/14 T2 Green's Functions and Variational Methods

Instructor: George Bluman
Office: Math Annex 1112
Email: bluman@math.ubc.ca
Office hours: by appointment (or drop-in if available)

Optional Texts:
Haberman, Applied Partial Differential Equations (Prentice-Hall)
Gelfand & Fomin, Calculus of Variations (Dover)

References:
Courant & Hilbert, Methods of Mathematical Physics, Volumes I & II (Interscience)
Duff & Naylor, Differential Equations of Applied Mathematics (Wiley)
Carrier & Pearson, Partial Differential Equations (Academic Press)
Copson, Partial Differential Equations (Cambridge)
Gustafson, Introduction to Partial Differential Equations and Hilbert Space Methods (Wiley)
Tikhonov & Samarskii, Equations of Mathematical Physics (Pergamon)
Zauderer, Partial Differential Equations of Applied Mathematics (Wiley-Interscience)
Goldstein, Classical Mechanics (Addison-Wesley)
Landau & Lifshitz, Mechanics (Addison-Wesley)
Bliss, Calculus of Variations (MAA monograph)
Weinstock, Calculus of Variations (Dover)
Bluman & Kumei, Symmetries and Differential Equations (Springer)
Bluman, Cheviakov & Anco, Applications of Symmetry Methods to Partial Differential Equations (Springer)—available online at no charge from the UBC Library

Topics:

I. Green's functions for linear differential equations 6 weeks

distributions (Dirac delta function, other distributions)
Green's functions for ODEs
Green's functions for PDEs
--homogeneous heat equation in one space dimension—infinite, semi-infinite and finite domains, fundamental solution, sources and sinks
--inhomogeneous heat equation in one space dimension
--adjoint operator
--Green's function for a Dirichlet problem
--Dirichlet problem for Laplace's equation
- Laplace's equation in two space dimensions—source solution, Dirichlet problems for a disc, for a semicircular region, for a half-plane, for an arbitrary domain (conformal mapping)
- Laplace's equation in three space dimensions—source solution

--Neumann problem for Laplace's equation
--linear elliptic equations (Dirichlet, Neumann and Robin problems)
- how to find formally Green's function for an arbitrary domain in terms of eigenvalues and eigenfunctions

--linear parabolic equations in n space dimensions for an arbitrary domain

--linear wave equations in *n* space dimensions for an arbitrary domain

- domain of dependence, range of influence, characteristic cone

II Calculus of variations 3.5 weeks

connections with classical mechanics—Lagrangian, path of least action

extremizing a functional

--normed linear space, admissible functions, first variation, necessary condition, Gâteaux variation, Euler derivative, Euler equation, fixed endpoints, free endpoints, brachistochrone problem

extremal problems with side conditions (constraints)

. --extremizing functions with function constraints; Lagrange multipliers

--extremizing functionals with functional and/or function constraints

- isoperimetric problem
- geodesics on a sphere

III Eigenfunction expansions 2.5 weeks

IV Rayleigh-Ritz and finite element methods 1 week

Pre-requisites: Math 217 or 227 or 317; 80% in Math 316 (257) or pass in Math 400

Grading: 30% from homework assignments, 20% from the midterm and 50% from the final exam.

There will be about 8 homework assignments (each will be due at the beginning of the class on the due date). Late homework will not be accepted. ***Copying solutions from another student, from the web or any other source and turning them in as your own is a violation of the Academic Code and will lead to severe punishment. However, you are encouraged to work with other students in the class.***

Likely, not all homework problems will be graded each week. Homework assignments and their solutions will be sent to you by e-mail.