MATH 360 — Mathematical Modelling in Science

2013W Term 1 September 2013

Tuesdays and Thursdays, 9:30 - 11:00 am, LSK 310

INSTRUCTOR:

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TEACHING ASSISTANT:

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GRADES AND EXAM DATES:

Quizzes (10%): based on reading assignment, in class every Thursday

Lab assignments (30%): due Fridays at 5 pm
Preferred method of submission: electronic (pdf) copy to Stilianos Louca, louca@zoology.ubc.ca
Option: Leave hard copy outside Rm 104 in the Biodiversity Research Centre

Midterm Exam I: Tuesday, October 15, 2013
Midterm Exam II: Thursday, October 17, 2013

The better of the two midterm grades counts for 30%.

The **final exam** counts for 30% (dates tba).

Course Outline: **Review of Math 360, Fall 2013**

Below are, very briefly, the main topics covered in the course. You should be familiar with these topics for the final exams. You should also be familiar with the methods used in the lab projects throughout the course.

1. Optimization in one variable

You should understand how first and second derivative of single-variable functions are used to determine maxima and minima, and how to use Matlab for finding derivatives, finding zeroes of a function, evaluating functions at a given point, and for plotting functions.

2. Discrete-time dynamical systems

The discrete-time dynamical system that we investigated are given by singlevariable functions F(x):

$$x(t + 1) = F((x(t)) . \qquad (1)$$

• You should understand how a function F(x) gives rise to a discrete-time dynamical system, and how the corresponding time series is obtained by choosing an initial condition x(0), and then repeatedly applying the function F to obtain the time series {x(0), x(1) = F(x(0)), x(2) = F(x(1)), ..., x(t + 1) = F(x(t)), ...}. You should be able to produce and plot such time series using Matlab.

• You should understand the method of cobwebbing as a way to visualize the time series resulting from the dynamical system (1). Conwebbing is based on a plot showing the graph the function F(x) as well as the diagonal y = x.

• Equilibrium points of the dynamical system (1) are given as solutions x* of the equation F(x*) = x*. The question of whether the dynamical system returns to an equilibrium after a small perturbation is addressed by the stability condition

|\frac{dF}{dx}(x*)| < 1. (2)

A stable equilibrium may be approached monotonically or via damped oscillations, depending on the sign of dF/dx(x*). You should have a good understanding of these basic issues.

• The dynamical system (1) may exhibit complicated dynamics, ranging from periodic dynamics (e.g. 2-cycle, 4-cycle, etc.) to chaotic dynamics. You should understand the defining feature of chaotic dynamics, sensitive dependence on initial conditions (the butterfly effect), and how this leads to unpredictability in real systems, such as the weather.

• You should understand how to read bifurcation diagrams for dynamical systems of the form (1). This means that you should understand that the yaxis in a bifurcation diagram shows the long-term behaviour of the system, and that a bifurcation diagram shows how this long-term behaviour changes as a function of the bifurcation parameter (which is on the x-axis). You should know what the period-doubling route to chaos is, and you should be able to produce bifurcation diagrams using Matlab.

3. Continuous-time dynamical systems

Continuous-time dynamical systems are given as differential equations, which describe the instantaneous rate of change of a dynamic variable.

• We have investigated single-variable differential equations of the form

$$\frac{dx}{dt} = f(x). \qquad (3)$$

Here $x(t)$ is the dynamic variable, whose trajectory over time one would like to know, and $f(x)$ is a single-variable function. For simple functions f, e.g. for linear functions, analytical expressions of $x(t)$ as a function of the time variable t can be found, e.g using the method of separation of variables. (If f is linear, the solution $x(t)$ is an exponential function.)

• Equilibrium points of (3) are given by solutions x^* of the equation $f(x^*) = 0$. The stability of an equilibrium x^* is determined by the sign of $df/dx(x^*)$. Systems of the form (3) cannot exhibit oscillatory dynamics. The dynamics of a differential equation of the form (3) can be understood by plotting the function $f(x)$.

• Differential equations can be obtained from “chemical reactions” between individual agents. These agents can be molecules, as in real chemical interactions, or they can be individual organisms undergoing birth and death events. You should understand how reactions between individual agents generate differential equations via the law of mass action. You should be able to use Matlab’s symbolic solver (dsolve command) to solve differential equations of the form (3).

• When there is more than one type of individual whose concentrations change over time, there is more than one differential equation describing the dynamics. Accordingly, we have studied systems of coupled differential equations of the form

$$\frac{dx}{dt} = f(x, y) \qquad (4)$$
$$\frac{dy}{dt} = g(x, y) \qquad (5)$$

Here $x(t)$ and $y(t)$ are the dynamic variables, $f(x, y)$ is a function of 2 variables describing the rate of the change in x, and $g(x, y)$ is a function of 2 variables describing the rate of the change in y, depending on the current state of the dynamic variables x and y.

• Equilibrium points (x^*, y^*) of the system of differential equations (4,5) are given as solutions to the two equations $f(x^*, y^*) = 0$ and $g(x^*, y^*) = 0$. You should be able to use Matlab to find such equilibrium points

• You should be able to use Matlab to solve systems of differential equations of the form (4,5) numerically (e.g. by modifying the files provided in lab 7).

4. Game theory

One particular class of one-dimensional differential equations occur in game theory. We have studied 2x2-games, i.e., games played between two players, each of which can have one of two strategies. Such games are given by payoff matrices, and their dynamics are described by a differential equation $dp/dt = f(p)$, where 0 is less than and equal to $p(t)$ less than and equal to 1 is the frequency of one of the strategies at time t (so that $1 - p(t)$ is the frequency of the other strategy). This equation is called the replicator equation. You should know the meaning of payoff matrices and how to interpret them, and you should know how to construct the replicator equation from a given payoff matrix.

5. Evolution in finite populations
You should understand the setup of the stochastic process describing evolution of two types in a finite population of fixed size. This process is based on assigning birth and death probabilities to individuals of both types. We have studied both neutral and non-neutral evolution in this context. You should know that the fixation probability is the probability that the stochastic process ends up with only type 1 individuals if the process is started with one type 1 individual and $N - 1$ type 2 individuals (where N is the population size). You should know what the fixation probability is in the neutral case.