

Ordinary Differential Equations

Special Section for Mech 2

Overview. This is a special section of MATH 255 dedicated to the MECH 2 program. This award-winning collaborative model for Engineering education involves a cohort of students who enjoy an intensive, integrated experience in which lectures, labs, and tutorials are closely coordinated. Details of the Mech 2 program are online at

<http://mech.ubc.ca/undergraduate-students/mech-2/>

Instructor. Professor Philip D. Loewen, office MATH 207, loew@math.ubc.ca. See also

<http://math.ubc.ca/~loew/mech2/>

Learning Activities. Professor Loewen will present 28 formal lectures and lead occasional scheduled Question-and-Answer (QA) sessions. In addition, there will be weekly small-group tutorials led by a Teaching Assistant. Weekly homework assignments and computer labs, offered in collaboration with the companion course MECH 221 (*Engineering Science I*), will also support student learning for many of the core concepts in the course.

Lecture Outline (Tentative).

1. Linear Interpolation in 1D and 2D; Richardson Extrapolation
2. Taylor Polynomials, with error analysis
3. Numerical differentiation, with O-notation for errors
4. Approximating definite integrals (Averaging, Trapezoidal Rule, Simpson's Rule)
5. Numerical Solution of Differential Equations — Euler and Improved Euler
6. Numerical Solution of Differential Equations — Runge-Kutta and ODE45
7. Linear Scalar Equations (cont): Integrating Factors
8. Separable Equations with applications: mixing, falling
9. Autonomous Equations (cont): Critical Points, Phase Line
10. Autonomous Equations (cont): Stability, Linearization
11. Scaling, Non-dimensionalization essential Engineering skills
12. Linear Time Invariant, Single-Input Single-Output: homogeneous solutions
13. Second Order Linear Equations (cont): Homogeneous Solutions for Complex Roots
14. Method of Undetermined Coefficients, a.k.a. "Guess and Check"
15. Mass Spring Systems; Electrical Analogies. Homogeneous Solutions
16. Resonance, Near Resonance and Beats — mechanical and electrical implications
17. Higher order equations; integral formula for particular solutions
18. First Order Linear ODE Systems; Phase Space; Eigenvalue connections
19. Linear Systems: Phase Plane Portraits (Real Eigenvalues)
20. Linear Systems (cont): Complex Eigenvalue Case
21. Beautiful unifying view of scalar and vector situations
22. Linear Systems (cont): Inhomogeneous Systems, esp resonance
23. Laplace Transforms (LT): introduction
24. Properties and calculations; Shift Theorem #1

25. LT (cont): solving DE's & transfer function
26. Steady-state response to sinusoidal input
27. Heaviside unit step; Shift Theorem #2
28. Generalized functions; Dirac delta function; applications

Textbook (Required). Brannan, James R., and William E. Boyce, *Differential Equations, An Introduction to Modern Methods and Applications*, 2/e. John Wiley & Sons, 2011.

Evaluation. Students grades will be determined by their demonstrated understanding on weekly homework assignments, weekly submissions in labs (both physical labs and computer labs), weekly quizzes, and two final examinations. MECH 2 is so deeply interdisciplinary that almost all of these activities will combine mathematical work with engineering analysis from a related subject. Therefore each student's grade in this section of MATH 255 will be the same as that student's grade in the 12-credit companion course MECH 221. Much more detail is provided in the *Mech 2 Handbook* and on the UBC Connect site dedicated to the program.