Math 609

Introduction to Random Schrödinger Operators

This course is an introduction to mathematical theory of random Schrödinger operators. These operators model an electron in a disordered solid. The goal is to understand the spectral and transport properties of these operators. A key concept is localization. An electron can move freely through a crystal where the atoms are arranged periodically, but in a disordered solid, where the atomic charges are distributed randomly, the electron can become trapped. This phenomenon was first explained by the physisict P.W. Anderson in 1958 and is now called Anderson localization. It took 20 years for the first mathematical proofs to appear. Today, there are two available methods for proving localization in all dimensions: the fractional moment method and multi-scale analysis. One dimension is special, and there are many additional tools available. This course will concentrate on the fractional moment method and a selection of one dimensional results. If time permits we will look at multi-scale analysis at the end of the course.

Instructor, Time and Location

Instructor: Richard Froese
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Hours: By appointment
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Time: Tuesday and Thursday 2:00-3:20
Location: MATH 204

This page
http://www.math.ubc.ca/~rfroese/math609
will be updated throughout the term.

Course Information

Problem sets

I will distribute problem sets periodically throughout the term. Your grade in the course will be based on these.

Prerequisites

Math 511 or 512 would be an asset, but the course will be accessible to students with a basic knowledge of Hilbert spaces (up to the spectral theorem for self-adjoint operators) and elementary probability. I will spend some time at the

beginning reviewing some needed background in the spectral and scattering theory of Schrödinger operators.

Resources

Books:

- Carmona and Lacroix, *Spectral Theory of Random Schrödinger Operators*, Birkhauser
- Cycon, Froese, Kirsch and Simon, *Schrödinger Operators, with applications to Quantum Mechanics and Global Geometry*, Springer. Chapter 9.
- Pastur and Figotin, *Spectra of Random and Almost-Periodic Operators*, Springer
- Stollman, *Caught by Disorder: Bound States in Random Media*, Birkhauser

Review articles and lecture notes on random Schrödinger operators:

- An Invitation to Random Schrödinger operators, by Werner Kirsch.
- An Introduction to the Mathematics of Anderson Localization., by Gunter Stoltz
- A Short Introduction to Anderson Localization, by Dirk Hundertmark
- Lectures notes by Vojkan Jaksic for a course in Random Schrödinger operators.

General references for Schrödinger operators

- Gustafson and Sigal, *Mathematical Concepts of Quantum Mechanics*
- Hislop and Sigal, *Introduction to spectral theory: with applications to Schrödinger operators*
- Simon, *Trace ideals and their applications*
- Outline of Quantum Mechanics: A set of notes by Bill Faris. Bill is visiting UBC this year, and I'm hoping we can give him some feedback on these.
- Introdution to Spectral Theory of Schrödinger Operators: Lecture notes by A. Pankov

Outline

1. Introduction to Random Schrödinger Operators

- Basic formalism of non-relativistic QM, discrete Schrodinger operators in Z^d and the Anderson model, definition of dynamical localization
- Review of spectral theorem, decomposition of the spectrum, RAGE theorem, min/max, Stone-Weierstrass
- Random potentials, almost sure spectrum for the Anderson model, ergodic potentials, almost sure ac/sc/pp spectrum.
- Density of states
- Wegner estimate
- Lifshitz tails

2. Localization via the fractional moment method

- Fractional moment bounds imply localization
- Proof of fractional moment bounds for large disorder
- Band edge localization

3. One dimension

- Simon-Spencer theorem
- Lyapunov exponent and Ishi-Pastur-Kotani theorem
- Kunz-Soulliard and localization at all disorders

4. Multiscale analysis

- Sketch of basic ideas