

**MATH 603: TOPICS IN TOPOLOGY  
GROUP ACTIONS ON  $\mathbb{R}$ -TREES  
2013 WINTER TERM 2**

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**Classroom:** Mathematics 103  
**Lecture:** Tuesday, Thursday 2-3:30pm

**Course Description**

Group actions on trees play an important role in geometric group theory and appear naturally in studying the *Teichmüller space* of a surface and its analogue, *outer space*, for a free group. We will begin with some basic properties of  $\mathbb{R}$ -trees and their groups of isometries. Topics will include the topology, compactification, and contractibility of Teichmüller space and outer space in the setting of  $\mathbb{R}$ -trees. We will aim to understand in particular some recent developments in understanding the boundary of the compactification of Culler-Vogtmann's outer space.

**Relevant Reading**

I recommend, and will be at times referring to, some of the following sources:

- M. Bestvina: Degenerations of the hyperbolic space, *Duke Math.*, Vol. 56, No.1, 1988.
- M. Clay: Contractibility of deformation spaces of  $G$ -trees, *Algebraic & Geometric Topology*, Vol. 5, pp. 1481–1503, 2005.
- M. Culler and J.W. Morgan: Group actions on  $\mathbb{R}$ -trees, *Proc. London Math. Soc.* (3) 55 pp. 571–604, 1987.
- M. Cohen and M. Lustig: Very small group actions on  $\mathbb{R}$ -trees and Dehn twist automorphisms, *Topology* Vol. 34, No. 3, pp. 575–617, 1995.
- M. Kapovich: *Hyperbolic manifolds and discrete groups*. Progress in Mathematics, 183. Birkhäuser Boston, Inc., Boston, MA, 2001.
- J. Morgan and P. Shalen: Free actions of surface groups on  $\mathbb{R}$ -trees, *Topology* Vol. 30, No. 2, pp. 143–154, 1991.
- F. Paulin: Topologie de Gromov équivariante, structures hyperboliques et arbre réels, *Invent. math.* 94, pp. 53–80, 1988.
- F. Paulin: The Gromov topology on  $\mathbb{R}$ -trees, *Topology and its Applications* 32, pp. 197–221, 1989.
- J. P. Serre: *Trees*. Translated from the French original by John Stillwell. Corrected 2nd printing of the 1980 English translation. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003.