

Introduction to Harmonic Analysis - Math 541

Fall 2012

- **Instructor:** *Malabika Pramanik*
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- **Office hours:** *To be announced.*

- **Web page:** The course website is

<http://www.math.ubc.ca/~malabika/teaching/ubc/fall12/math541/index.html>

Homework assignments and all relevant course information (such as changes to office hours if any, or solutions to homework problems if needed) will be posted here.

- **Text:** There are no required textbooks. The following textbooks are recommended.
 - *Lectures on Harmonic Analysis (2003)*, by *T.H. Wolff*, AMS, ISBN: 978-0-8218-3449-7.
 - *An Introduction to Harmonic Analysis (3rd edition)*, by *Y. Katznelson*, Cambridge, ISBN: 978-0-521-54359-2.
 - *Singular Integrals and Differentiability Properties of Functions (1970)*, by *E. Stein*, Princeton University Press, ISBN: 0-691-08079-8.
 - *Introduction to Fourier Analysis on Euclidean Spaces (1971)*, by *E. Stein*, and *G. Weiss*, Princeton University Press, ISBN: 0-691-08078-X.
 - *Harmonic Analysis: Real-variable Methods, Orthogonality and Oscillatory Integrals (1993)*, by *E. Stein*, Princeton University Press, ISBN: 0-691-03216-5.
 - *Classical and Modern Fourier Analysis*, by *L. Grafakos*.
- **Course outline :** The core topics of the course are the following:
 1. *Basic material concerning Fourier series, Fourier transform and Fourier inversion*
 - Fourier basis for $L^2(\mathbb{T})$
 - Convolution
 - Approximate identities
 - Temperate distributions
 - Some applications
 2. *Convergence of Fourier series*
 - Decay of Fourier coefficients
 - Uniform convergence of Fourier series

- Pointwise convergence and almost everywhere divergence
- Norm convergence

3. *Interpolation of operators*

- Complex methods (Riesz-Thörin theorem, analytic interpolation)
- Real methods (Marcinkiewicz interpolation theorem)
- Applications (Hausdorff-Young inequality, Young's convolution inequality, fractional integration, Hardy-Littlewood maximal theorem).

4. *Singular integral operators*

- Calderón-Zygmund kernels
- Some multiplier operators
- The Calderón-Zygmund decomposition
- L^p boundedness of Calderón-Zygmund singular integral operators
- Homogeneous distributions, Hilbert transform, Riesz transform.

5. *Littlewood-Paley theory*

- Almost orthogonality in Hilbert spaces, Cotlar-Knapp-Stein lemma
- A square function that characterizes L^p
- Variations and applications

Time permitting, we will also consider other special topics.

- **Lectures :** Monday, Wednesday, Friday 2 pm - 3 pm in Mathematics 202.
- **Grading Policy :** To be announced.