UNIVERSITY OF BRITISH COLUMBIA

Math 428/513 Sep-Dec 2012

Mathematical Classical Mechanics

Description:

- In this course we will follow Arnold's book "*Mathematical methods of Classical Mechanics*" and cover Newtonian Mechanics, Constraint Free Lagrangian Mechanics, Mechanics on Manifolds, and Differential Forms (Introduction to Hamiltonian Mechanics).
- Many different mathematical methods and concepts are used in classical mechanics: differential equations and phase flows, smooth mappings and manifolds, Lie groups and Lie algebras, symplectic geometry and ergodic theory. Many modern mathematical theories arose from problems in mechanics and only later acquired the axiomatic-abstract form. This course will help us to gain a unified perspective.
- This course is intended to complement classical mechanics courses like Physics 206 in the sense that the physical background will be developed but the emphasis will be on the resulting mathematical analysis.

Prerequisites: Students should already have some experience with rigorous mathematics (like Math 320 and 321) and with classical mechanics (like Physics 206) although these prerequisites may be waived at the discretion of the instructor.

Textbook: V. I. Arnold, Mathematical Methods of Classical Mechanics (Graduate Texts in Mathematics), second edition, translated by A. Weinstein and K. Vogtmann, Springer-Verlag.

A few other references I will consult:

1. L. D. Landau and E. M. Lifshitz: Mechanics, third edition
2. J. B. Marion and S. T. Thornton, Classical dynamics of particles & systems
3. H. Goldstein, Classical mechanics, second edition

Grading:

1. There will be weekly problem sets accounting for 50% of the final mark.
2. The final exam will account for 50% of the final mark.
3. Grades will probably be scaled.

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homepage: `http://www.math.ubc.ca/~ttsai/courses/428-513-12F/`

Outline:

1. Newtonian Mechanics
   (a) The principles of relativity and determinacy, the galilean group, Newton's equations
   (b) Examples: the harmonic oscillator, pendulum and central fields
   (c) An introduction to phase space, conservation of energy, momentum and angular momentum
2. Constraint Free Lagrangian Mechanics
   (a) Variational problems and the Euler-Lagrange equation
   (b) The lagrangian and Hamilton's principle of least action
   (c) The hamiltonian and Hamilton's equations
   (d) Liouville's theorem
   (e) Poincaré recurrence theorem
3. Lagrangian Mechanics on Manifolds
   (a) The introduction of manifolds through constraints
   (b) Differentiable manifolds and tangent bundles
   (c) Lagrangian dynamics
   (d) Symmetry and Conservation laws: Noether's theorem
4. Differential Forms
   (a) Exterior algebra, differential forms on manifolds, exterior differentiation, vector analysis
   (b) Chains, integration of differential forms
   (c) Stokes' theorem
   (d) Poincaré lemma

   The last topic is the first chapter of Part III Hamiltonian mechanics, on which I will give an overview.