Math 223: Linear Algebra
Fall Term, 2012

Lior Silberman

v1.0 (September 5, 2012)

Course Website	`http://www.math.ubc.ca/~lior/teaching/1213/223_F12/`
Contact me at	MAT 229B — 604-827-3031 – `lior@math.ubc.ca`
My Website	`http://www.math.ubc.ca/~lior/`
Class	MWF 10:00-11:00, MATX 1100
Office Hours	M 14:00-15:30, Th 14:00-15:00 or by appointment
Textbook	Friedberg, Insel, and Spence, *Linear algebra* (4th edition)
Course Prerequisites	One of: (a) MATH 121 or (b) score$\geq$ 68% in Math 101, MATH 103, MATH 105, SCIE 001

Note: the prerequisite may be waived for exceptional incoming students.

About the course

We will explore linearity, one of the simplest mathematical phenomena, and Linear algebra, a basic mathematical language. It is used within mathematics, for example to talk about differential equations and strategies in game theory, and generally within science to describe the world around us. For example, it is the most natural language to express quantum mechanics, is basic to studying signal processing and underlies calculations in computer graphics. We will emphasize mathematical ways of thought, including understanding definitions and writing rigorous proofs.

> **This course is aimed at excellent students (typically honours students, though anyone may enroll). It is more abstract and covers more material than MATH 152 and MATH 221.**

The textbook for this course is *[2], Linear algebra* by Friedberg, Insel and Spence. It will serve as a reference and as a source of practice problems. That said, the material is standard and is covered by many textbooks. Purely as examples note [4] (emphasis on worked problems). and [3] (extremely abstract treatment)

There will be three parallel themes through the course: computation, abstract linear algebra, and general mathematical thinking. Exams will consist roughly of 30-40% computation and 60%-70% abstract material.

Teaching and learning

Course plan ("material")

- Vector spaces
    - Vector spaces and subspaces

  - Linear dependence and independence
  - Bases and dimension
  - Systems of linear equations
- Linear transformation
  - Linear functionals and linear maps
  - Geometry of linear transformations
  - Kernel and image
  - Representation by matrices
  - Composition and matrix multiplication
  - Invertibility
- Determinants
- Eigenvalues
  - Eigenvalues and eigenvectors
  - The characteristic polynomial
  - Diagonalization
  - Complex eigenvalues
- Inner-product spaces
  - Inner products
  - Symmetric and self-adjoint maps
  - The spectral theorem

Learning goals

By the end of the course, you will be expected to exhibit:

- Computational skills, including
  - Vector and matrix arithmetic, including multiplication of matrices and determination of inverses;
  - Recognizing and solving systems of linear equations;
  - Transforming matrices by column and row reduction;
  - Deciding whether sets of vectors are dependent or indepdent, including computing spans and deciding membership in subspaces;
  - Deciding whether linear maps and matrices are invertible;
  - Computing determinants by several techniques.;
  - Recognizing eigenvalue problems and computing eigenvalues and corresponding eigenvectors;
  - Computing in inner product spaces, including finding orthogonal complements.
- Language skills, including
  - Convert statements in mathematics and science to the language of linear algebra

- Use the vocabulary of linear algebra to discuss these statements.

- Abstract algebra skills, including
  - State basic definitions (vector space; linear transformation; eigenvector; and so on)
  - Prove simple statements involving the definitions.
  - Decide whether sets are vector spaces, and whether subsets are subspaces. Decide whether maps are linear transformations.
  - Construct vector spaces from other vector spaces by various means.
  - Prove statements about abstract sets.

- Mathematical skills, including
  - Solving problems where the method of solution has not been given in advance.
  - Applying mathematical technique to convert solution ideas to rigorous solutions.
  - Constructing proofs of given statements, especially in the context of linear algebra.
  - Writing clear and concise proofs.
  - Recognizing whether given proofs are correct or incorrect.

What you can expect from me

- Demanding homework and examinations.
- Various approaches to the material including lecturing, classroom discussion and in-class assignments.
- Responses to your questions and concerns: continuously in class and during my office hours, within reasonable time by e-mail outside class.
- Timely and clear explanations of what is correct in your work and what is not, and how you can improve.

What's expected from you

- Come prepared to class, having read the relevant material in the textbook. Information will be posted on the course website.
- Actively participate in the course: do the reading, think about the material, solve the problem sets, and ask questions.
  - Asking questions when you don't understand, or want to learn more, ensures that you get what you want out of the course. Ask me questions in class, by email, and during office hours. Also, ask your colleagues questions outside of class – you will both benefit from the discussion!
  - Working on the problem sets is *absolutely essential* for learning the material. **It is extremely rare for students who skip problem sets to do well on exams.**
  - I may call on you in class.
- Submit written work that is readable and communicates your ideas.
  - Reasoning needs to be conveyed in English sentences, not as a sequence of formulas.

Official Policies

General policies

- For every week after the first, there will be assigned reading (usually from the textbook). The discussions in class and the on-line homework will assume that you have read these chapters beforehand.
- **Late or missed exams and assignments will not be accepted for credit and will be given a grade of zero**. In exceptional circumstances (a proof of the emergency is required, and advance notification if possible will be required) the missed work will be registered (and not count toward the average of that part of the course) if you finish it and hand it in after the emergency has passed.
- All assertions require ***proof*** unless the problem states otherwise. No matter the operative word ("find", "solve", "establish", "calculate", "determine" ...), you must justify your answer.
- Written work should be presented carefully, in complete English sentences, and with sufficient detail. A "correct sequence of formulas" will only merit partial credit. Examples of the expectations will be distributed together with the first problem set.

Homework

- There will be up to twelve problem sets posted to the course website, due at the *beginning* of class on the day shown. I will drop the lowest score when calculating the homework grade.
  - Problems will focus on conceptual material, with some calculational problems.
  - You are encouraged to work on solving the problems together. However, each of you must write your solutions independently, in your own words. You may (and should) share your ideas but you may not share your written work.
  - It is possible that only certain problems from a problem set will be selected for grading. Complete solutions will be posted in any case.
  - Solutions will be posted on the secure (Connect) website.
- There *may be* a few additional computational exercize sets, to be completed online using the WeBWork system. These will be announced well in advance.
  - The problems will focus on technique.
  - Solutions may not be posted.

Exams

- There will midterm exams in class on Friday, October 5th and on Wednesday, November 7th.
  - If you need special accommodations when taking written exams, please contact the Office of Access & Diversity (`access.diversity@ubc.ca`).
  - If a midterm (or final) exam conflicts with a religious observance, please contact me *at least two weeks ahead of time* so we can make appropriate arrangements..
- There will be a final exam during the usual exam period.

Final grade

- The final grade will be calculated as follows:

Problem sets: 20%
Midterms: 15% each
Final exam: 50%

References

[1] Axler, *Linear algebra done right*

[2] Friedberg, Insel, and Spence, *Linear algebra*

[3] Halmos, *Finite-Dimensional Vector Spaces*

[4] Lipschutz, *Schaum's Outline of Linear Algebra*

The material of the course is standard; any textbook titled "linear algebra" will cover the computational aspects, and most will discuss the abstract algebra aspects as well.