

ADDITIVE NUMBER THEORY - MATH 613, Fall 2011

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Prerequisites: Some familiarity with analytic number theory, such as the Prime Number Theorem is useful.

Suggested Texts:

We will provide notes to some of the topics discussed below. Additional suggested readings:

Terence Tao, Van Vu: *Additive Combinatorics* Cambridge (2006)

K. Soundarajan's lecture notes, <http://math.stanford.edu/~ksound/Notes.pdf>

B. Green and T. Tao: *The primes contain arbitrary long arithmetic progressions*

B. Green, T. Tao and T. Ziegler: *Linear equations in the primes*

Course Material and Topics: The course will provide an introduction to some of the recent remarkable results in additive number theory/combinatorics. This is a new rapidly developing area that draws upon techniques from different fields; analytic number theory, harmonic analysis, combinatorics (and even ergodic theory). We will start with basic discrete Fourier analysis and move quickly towards active research topics.

The topics will include the following:

I. Roth's theorem on 3-term arithmetic progressions.

We will prove via Fourier analysis that a subset of the natural numbers of positive upper density contains (many) 3-term arithmetic progressions. At the same time we introduce some basic notions such as Bohr sets, pseudo-randomness, density increments and so on.

II. van der Waerden's 3-prime theorem.

We will find the asymptotic number of 3-term arithmetic progressions consisting of prime numbers up to a given large number N . This is a result due to van der Waerden (it is essentially "equivalent" to the ternary Goldbach problem proved by Viogradov). We will prove it from the point of view of the recent spectacular result of Green-Tao-Ziegler on prime solutions of linear equations. We will introduce some of the basic notions and ideas in their proof, such as the W-trick to find the local factors,

the Goldston-Yildirim "sieve", and the asymptotic orthogonality of the Mobius function to linear phase functions.

III. Gowers proof for 4-term arithmetic progressions.

We will give the complete details of Gower's Fourier analytic proof on the existence of 4-term arithmetic progressions in sets of positive density. First we discuss Gowers norms, higher degree uniformity and the so-called generalized von Neumann inequality which shows that the Gowers norms control "linear patterns" in sets of positive density. Then we discuss combinatorial methods; the Balog-Szemerédi theorem, and elements of Ruzsa's proof of Freiman's theorem needed in the proof.

IV. The Green-Tao theorem on arithmetic progressions in the primes.

We will discuss the main elements of the proof, introduce the key transference principle (Gower's approach), the pseudo-random majorant, the linear forms and correlations. If time permits we give the details of the proof assuming the validity of Szemerédi's theorem for general k -term arithmetic progressions.