THE UNIVERSITY OF BRITISH COLUMBIA

Greg Martin

Professor
Department of Mathematics

Lectures: Mondays, Wednesdays, and Fridays, 12:00–12:50 PM, room MATH 204 (Mathematics building)
Office hours: by appointment or drop in
Office: room MATH 212 (Mathematics Building)
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Course description: This course covers the fundamental techniques in classical analytic number theory. The objects of study are the natural numbers; the theorems sought are statistical statements about the distribution of primes, the number of divisors of integers, and similar multiplicative questions; the techniques involve both "by hand" real analytic estimation and contour integration of meromorphic functions. The successful student will be well equipped to understand much of the current research literature in this area.

Prerequisites: Students should have had a previous course in number theory (preferably MATH 537 here at UBC). It will be assumed that the student has had the usual undergraduate training in analysis (for example, MATH 320) and a strong course in complex analysis (preferably MATH 508). In particular, in complex analysis students should have a working knowledge of the residue theorem, logarithmic derivatives, and the argument principle. Students will also need to have a working knowledge of LaTeX, although this can be acquired along the way if necessary.

Course textbook: This course will require the book by Montgomery and Vaughan, *Multiplicative Number Theory I: Classical Theory* (Cambridge University Press, 2006). Please let me know if you encounter problems buying the textbook from the UBC bookstore. Here is a link to Hugh Montgomery's home page, at which you can access a list of errata for the book. If you find an error not on the list, you should email him!

Topics to be covered in this course:

- Dirichlet series and the Mellin transform
- Arithmetical functions and their summation and estimation
- Prime counting functions and Chebyshev's and Mertens's estimates
- The Riemann zeta function and its zeros
- The prime number theorem and applications
- Dirichlet characters and *L*-functions
- The prime number theorem in arithmetic progressions

Evaluation: The course mark will be based on eight homework assignments, due approximately every five class days (a little more often than once every two weeks), as well as group work done in class. Your homework will be marked on correctness, completeness, rigor, and elegance. A correct answer will not earn full marks unless it is completely justified, in a rigorous manner, and written in a logical sequence that is easy to follow and confirm. *Survival tip*: don't start these assignments the night before they're due! Anecdotal evidence suggests that each assignment could take as much as 15 hours or more to complete.

Other than the group work, *no handouts will be distributed in class*. All homework assignments and any other course materials will be posted below on this course web page. **Homework solutions must be prepared in LaTeX** and submitted to me in PDF format via email; please add your name to the filename before submitting your homework (for example, GregMartin-homework1.pdf). I will supply LaTeX templates with each assignment. All homeworks are due before the beginning of class (11:59 AM) on the indicated days.

- Homework #0: due Wednesday, January 10. Download both the TeX file and the PDF file.
- Homework #1: due ...

- Homework #2: due ...
- Homework #3: due ...
- Homework #4: due ...
- Homework #5: due ...
- Homework #6: due ...
- Homework #7: due ...

- Group Work #1: ...
- Group Work #2: ...
- Group Work #3: ...
- Group Work #4: ...
- Group Work #5: ...
- Group Work #6: ...
- Group Work #7: ...
- Group Work #8: ...
- Group Work #9: ...
- Group Work #10: ...

Students are allowed to consult one another concerning the homework problems, but *your submitted solutions must be written by you in your own words*. Students can be found guilty of plagiarism if they submit virtually identical answers to a question, or if they do not understand what they have submitted.

Because there are no exams, the lectures will continue into the beginning of the final exams period, ending on (probably) Friday, April 20.

Other possible references for analytic number theory:

- H. Davenport, *Multiplicative Number Theory*
- T. M. Apostol, *Introduction to Analytic Number Theory*
- H. Iwaniec and E. Kowalski, *Analytic Number Theory*
- P. T. Bateman and H. G. Diamond, *Analytic Number Theory: An introductory course*

Possible references for elementary number theory:

- I. Niven, H. S. Zuckerman, and H. L. Montgomery, *An Introduction to the Theory of Numbers*
- G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*