Greg Martin

Professor
Department of Mathematics

Contact Info	**Research and Papers**	Teaching	**Putnam**	**Curriculum Vitae**	**Other Math Links**	**Non-math Links**

Teaching

MATH 437/537

• Piazza

MATH 101

Past courses

My students

Other Links

UBC Number Theory group
NT seminar schedule
(edit number theory events)

UBC Math Department
Math faculty directory

Lectures: Mondays, Wednesdays, and Fridays, 10-11 AM, room MATH 105 (Mathematics building)
Office hours: through **Piazza**, or by appointment
Office: room MATH 212 (Mathematics Building)
Email address: use **Piazza** instead, or gerg@math.ubc.ca

Course description: The first few weeks will be spent quickly covering the foundations of elementary number theory: divisibility, congruences, prime numbers, and so on, some of which might already be familiar to you. Emphasis will be on a level of mastery sufficient for you to teach the material. Once we have this foundation, we will move on to roots of polynomial congruences, arithmetic and multiplicative functions, binary quadratic forms, and parametrizing Pythagorean triples. The two most important topics of the course are primitive roots and quadratic reciprocity. Topics that might also be covered include simple Diophantine equations, Diophantine approximation, and continued fractions.

Course textbook: *An Introduction to the Theory of Numbers*, by Niven, Zuckerman, and Montgomery, 5th edition (required). We will cover roughly the following sections:

- Sections 1.2–1.3: divisibility, gcds, primes
- Sections 2.1–2.3 and 2.6–2.8: congruences, solutions of congruences, Chinese remainder theorem, prime power modulus, prime modulus, primitive roots and power residues
- Sections 3.1–3.6: quadratic residues, quadratic reciprocity, Jacobi symbol, binary quadratic forms, their equivalence and reduction, sums of two squares
- Sections 4.2–4.4: arithmetic functions, Möbius inversion, recurrence functions
  - (if time permits: A.4, linear recurrences)
- Sections 5.1 and 5.3: solving $ax + by = c$ in integers, Pythagorean triangles
  - (if time permits: 5.2 and 5.4, simultaneous linear equations and assorted examples)
- Sections 6.1–6.3: Farey sequences, rational approximations, irrational numbers (as time permits)
- Sections 7.1–7.8: the Euclidean algorithm, uniqueness, infinite continued fractions, (best possible) approximations of irrationals by rationals, periodic continued fractions, Pell's equation (as time permits)

Notes to undergraduates: For all practical purposes, *MATH 437 is an honours course*! It treats roughly the same material as MATH 312 and 313 combined. Note that a student cannot have credit for both MATH 312 and MATH 437, nor for both MATH 313 and MATH 437. To enroll in MATH 437, an undergraduate student must have already taken, or be taking simultaneously, one of MATH 320 or MATH 322.

The word "elementary" in the title does not mean the course isn't difficult; rather it means that the course doesn't use techniques from real or complex analysis or from abstract algebra. The course will not require any particular background in number theory. What is required is "mathematical sophistication", which certainly includes being able to understand and write proofs. Be forewarned that this course will be taught at the level of a graduate course. Honours students typically will be well-equipped to succeed in this course.

Use of the internet: We will be using **Piazza** for all class-related announcements and discussion. Piazza is a question-and-answer platform specifically designed to expedite answers to your questions, using the

collective knowledge of your classmates and instructor. It has several features that facilitate discussion of mathematics, most notably LaTeX support. *You are encouraged to answer your classmates' questions, or to brainstorm towards answers, every bit as much as you are encouraged to ask questions.*

Evaluation: The course mark will be based on six homework assignments (75% of the final mark), due approximately every two weeks, and one final exam (25% of the final mark) with a format yet to be

approximately every two weeks, and one final exam (20% of the final mark) with a format yet to be determined. In the case of extreme disparity between the homework and exam marks, the instructor may use his discretion in assigning a final course mark.

Your homework will be marked on correctness, completeness, rigor, and elegance. A correct answer will not earn full marks unless it is completely justified, in a rigorous manner, and written in a logical sequence that is easy to follow and confirm. I plan on being pedantic about completeness of solutions (for example, if you invoke Euler's theorem to assert that $a^{\varphi(q)} \equiv 1 \pmod{q}$, you need to explicity acknowledge the fact that a must be relatively prime to q). Part of the goal of this course is to provide training and practice at writing proofs with sufficient rigor to be accepted by research journals.

Students are allowed to consult one another concerning the homework problems, but *your submitted solutions must be written by you in your own words*. If two students submit virtually identical answers to a question, both can be found guilty of plagiarism.

No handouts will be distributed in class. All homework assignments and any other course materials will be posted on Piazza in PDF format. I encourage you to ask questions on Piazza any time you're working towards understanding a concept; you can even do so anonymously if necessary. Part of the reason I don't have regularly scheduled office hours is that many questions can be answered through Piazza; in fact, I prefer using Piazza instead of email for questions related to the course (Piazza allows you to ask privately if necessary).

Homework solutions must be prepared in LaTeX and submitted in PDF format via Piazza. I will supply LaTeX templates with each assignment. All homeworks are due before the beginning of class (9:59 AM) on the indicated days.

- Homework #1: due Wednesday, September 17
- Homework #2: due Wednesday, October 1
- Homework #3: due Friday, October 17
- Homework #4: due Friday, October 31
- Homework #5: due Friday, November 14
- Homework #6: due Friday, November 28