

# MATH 517 PARTIAL DIFFERENTIAL EQUATIONS II

*Term 2 (Jan-April 2018)*

<http://www.math.ubc.ca/~jcwei/MATH517-2018.html>

**Instructor:** Juncheng Wei, Tel. 604-822-6510, E-mail: [jcwei@math.ubc.ca](mailto:jcwei@math.ubc.ca)

**Topic prerequisites:** Introduction to PDE (516 equivalent) or Han-Lin's book or Gilbarg-Trudinger's book (Chapter 1 to Chapter 9)

**Description :** This is a continuation of MATH516 (Elementary PDEs). In this course, we will continue a little bit of regularity theories such as De Giorgi's techniques, Evans-Krylov theory, etc. Then we will discuss variational and nonlinear techniques for nonlinear elliptic equations, such as Mountain-Pass-Lemma, Concentration-Compactness, Method of Moving Planes, Crandall-Rabinowitz bifurcation, Monotonicity Formula, De Giorgi's conjectures and infinite dimensional Liapunov-Schmidt reduction method.

- Review of regularity theory
- Regularity theory: De Giorgi's techniques ([HL])
- Topological fixed point theorems ([GT])
- Direct minimizations; Nehari manifolds ([S])
- Mountain-Pass-Lemma ([S])
- Maximum Principle and the Method of Moving Planes
- Concentration-Compactness
- Crandall-Rabinowitz bifurcation theorems and applications
- Liouville and De Giorgi type Theorems ([QS])
- Monotonicity Formula and its Applications

- Gluing Method I: Finite-dimensional reduction (subcritical case) ([W])
- Gluing Method II: Finite-dimensional reduction (critical case) ([W])
- Gluing Method III: Infinite-dimensional reduction (non-resonance case) ([W])
- Gluing Method IV: Infinite-dimensional reduction (resonance case) ([W])
- Gluing Method V: gluing method for parabolic blow-ups ([W])

**References:**

1. M. Struwe, Variational Methods, applications to nonlinear partial differential equations and Hamiltonian systems
2. Gilbarg-Trudinger, Elliptic partial differential equations of second order
3. Han-Lin: Elliptic partial differential equations
4. Quittner-Souplet: Superlinear parabolic problems, blow-up, global existence and steady-states
5. Wei: Introduction to gluing methods