UBC Mathematics 402 – Fall term 2019
Instructor: Dr. Samer Dweik, ESB 4112
Course supervisor: Dr. Nassif Ghoussoub, ESB 4144

Overview: This course deals with problems from geometry and mechanics that require minimization of functionals on infinite dimensional spaces. The classical theory, where minimization occurs on space of one-dimensional paths -as developed from Bernoulli through Euler to the present- will be the main focus.

Selected suggestions for readings:

- Lecture notes by Professor Philip Loewen http://www.math.ubc.ca/~loew/m402/
- Introduction to the Calculus of Variations, by Hand Sagan
- The Calculus of Variations, by Bruce van Brunt
- A Primer on the Calculus of Variations and Optimal Control, by Mike Mesterton-Gibbons
- G. A. Bliss, Calculus of Variations, Carus Mathematical Monographs, Chicago, 1925
- O. Bolza, Lectures on the Calculus of Variations, University of Chicago Press, Chicago, 1904

Prerequisite: C+ or better in either Math 320 or Math 301, or consent of the instructor.

Grading: 50% for the term (bi-weekly homework, one test), 50% for the final exam.

1. The Basic Problem of the Calculus of Variations
    - Typical Problems
    - Formulation of the general problem
    - Issues of existence of optimal curves, regularity, uniqueness

2. The First Necessary conditions:
    - The Fundamental Lemma of the Calculus of Variations
    - The Euler-Lagrange Differential Equation
    - The Weierstrass-Erdmann Corner Conditions

3. The Second-Order Necessary Conditions:
    - The Legendre Necessary Condition
    - Jacobi's Necessary Condition: Conjugate points
    - The Weierstrass Necessary Condition

4. Sufficient Conditions for the Basic Problem:
    - Convexity in the Variational Problem
    - Fields of extremals
    - The Hilbert Integral
    - Fundamental Sufficient Results

5. Modifications of the Basic Problem:
    - The Free Endpoint Problem
    - Variational Problems with Constraints: Isoperimetric Problem

6. The Simplest Optimal Control Problem:
    - The Value Function
    - Dynamic Programming Principle
    - The Hamilton-Jacobi Equation