Math 412: Advanced Linear Algebra
Fall Semester, 2019

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Course Website	`http://www.math.ubc.ca/~lior/teaching/1920/412_F19/`
Contact me at	MATX 1112 — 604-827-3031 – lior@math.ubc.ca
My Website	`http://www.math.ubc.ca/~lior/`
Class	TTh 14:00-15:30, GEOG 147
Office Hours	TBA
Textbook	None required; see below.
Course Prerequisites	Score$\geq$ 68% in both of MATH 320, MATH 322.

About the course

This is a second course in linear algebra, intended for students with some mathematical experience. Its runing theme will be the idea of *algebraic constructions*. After learning about to make such constructions we will apply some of them to construct the *Jordan canonical form* (the basic structure theory for linear maps). We will then discuss some ideas of analysis with matrices.

The material for this course is mostly standard, and appears in many references. Nearly all is covered in Roman's and Halmos's excellent books [6, 3], as well as in other standard textbooks (for example in [2], or [5]). References for analysis on matrices include [4] and Chapter 1 of [1].

Teaching and learning

Significant prerequisites

- Basic linear algebra at the level of Math 223: vector spaces, linear maps, Gaussian elimination and linear equations, determinants, eigenvalues and eigenvectors, inner product spaces.
- Group theory from Math 322: The group of invertible matrices, quotient groups, group homomorphisms.
- Analysis from Math 320: Convergence in $\mathbb{R}^n$; uniform convergence of functions; power series.

Course plan ("material")

(for a detailed schedule see the course website)

- Review of basic linear algebra
- New constructions

  - Direct sum and direct product
  - Spaces of homomorphisms and duality
  - Quotient vector spaces
  - Multilinear algebra: Tensor products
- Structure theory for linear maps
  - Matrix decompositions
  - The minimal polynomial and the Cayley–Hamilton Theorem
  - The Jordan canonical form
- Analysis with vectors and matrices
  - Norms on vector spaces
  - Operator norms
  - Matrices in power series: e^{tX} and its friends

What you can expect from me

- Demanding homework and examinations.
- Responses to your questions and concerns: continuously in class and during my office hours, within reasonable time by e-mail outside class.
- Clear explanations of what is correct in your work and what is not, and how you can improve.

What's expected from you

- Come prepared to class, having read the material specified on the course website in a textbook of your choice.
- Actively participate in the course: do the reading, think about the material, solve the problem sets, and ask questions.
  - Asking questions when you don't understand, or want to learn more, ensures that you get what you want out of the course. Ask me questions in class, by email, and during office hours. Also, ask your colleagues questions outside of class – you will both benefit from the discussion!
  - Working on the problem sets is *absolutely essential* for learning the material. **It is extremely rare for students who skip problem sets to do well on exams.**
  - I may call on you in class.
- Submit written work that is readable and communicates your ideas.
  - Reasoning needs to be conveyed in English sentences, not as a sequence of formulas.

Official Policies

General policies

- **Late or missed exams and assignments will not be accepted for credit and will be given a grade of zero**. In exceptional circumstances (a proof of the emergency is required, and advance notification if possible will be required) the missed work will be registered (and not count toward the average of that part of the course) if you finish it and hand it in after the emergency has passed.
- All assertions require ***proof*** unless the problem states otherwise. No matter the operative word ("find", "solve", "establish", "calculate", "determine" ...), you must rigorously justify your answer.
- Written work should be presented carefully, with sufficient detail in complete English sentences. A "correct sequence of formulas" will only merit partial credit. Examples of the expectations may be distributed together with the first problem set.
- I may designate material (e.g. definitions) for self-study, in which case you are responsible for learning this material before it is used in class and in problem sets.

Homework

- There will be up to eleven problem sets posted to the course website, due at the *beginning* of class on the day shown. I will drop the lowest score when calculating the homework grade.
  - Problems will focus on conceptual material, with some calculational problems.
  - You are encouraged to work on solving the problems together. However, each of you must write your solutions independently, in your own words. You may (and should) share your ideas but you may not share your written work.
  - It is possible that only certain problems from a problem set will be selected for grading.
  - Solutions will be posted on the secure (Canvas) website.

Exams

- There will one midterm exam in class, currently scheduled for Thursday, October 24th.
  - If you need special accommodations when taking written exams, please contact the Centre for Accessibility.
  - If the midterm (or final) exam conflicts with a religious observance, or if you have any other legitimate conflict, please contact me *at least two weeks ahead of time* so we can make appropriate arrangements.
- There will be a final exam during the usual exam period.

Final grade

- The final grade will be calculated as follows:

Problem sets: 30%
Midterm: 20%
Final exam: 50%

UBC boilerplate

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious, spiritual and cultural observances. UBC values academic honesty and students ae expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available herehere.

References

[1] Coleman, *Calculus on Normed Vector Spaces*

[2] Dummit and Foote, *Abstract Algebra*

[3] Halmos, Finite-Dimensional Vector Spaces

[4] Higham, *Functions of Matrices*

[5] Lang, *Linear Algebra*

[6] Roman, *Advanced Linear Algebra*

(References with hyperlinks may be freely downloaded from the publishers' websites while on the campus network).