Probability in high dimensions (Math 608): Detailed outline

This is a tentative outline. A running application of interest is compressed sensing. I hope to also include machine learning theory and to touch upon deep learning.

1. Behaviour of sums of random variables, non-asymptotic deviation inequalities
   a. Sub-Gaussian random variables
   b. Sub-exponential random variables
   c. Bernstein inequality
2. Concentration of measure
   a. Concentration on the sphere and in Gauss space.
   b. Implication: Lipschitz functions concentrate around their median.
   c. Application: Johnson-Lindenstrauss lemma
3. Non-asymptotic random matrix theory and extrema of stochastic processes
   a. Review of asymptotic random matrix theory (this is given for an intuition, key results will be stated without proofs)
   b. Connection of random matrices to stochastic processes
   c. Covering arguments
   c. Slepian inequality, Gordon inequality
   d. Matrix Bernstein inequality
   e. Application: Covariance estimation
   f. Involved covering arguments: Dudley inequality, Generic chaining
   g. Application in compressed sensing: Restricted isometry property for sub-sampled Fourier transform
   h. Majorizing measures theorem
   i. Conditioning of a random matrix restricted to a fixed set, with applications to convex programming
   j. Fano’s inequality (without information theory), showing optimality of compressed sensing results
4. Geometric functional analysis
   a. Gordon's escape through the mesh theorem, with application in compressed sensing

b. Random vectors drawn from convex bodies

d. Dvoretzky Milman theorem

d. Sudakov inequality

e. Low M* estimate

f. Sections of l_1 ball

5. More applications

a. Matrix completion

b. Machine learning (VC dimension, Rademacher complexity)

c. Deep learning? Maybe. It depends on class interest and time constraints.